

OPTIMUM BUCKLING DESIGN OF CYLINDRICAL STIFFENER SHELL UNDER EXTERNAL HYDROSTATIC PRESSURE

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ABSTRACT

This paper present an investigation of the collapse load in cylinder shell under uniform external hydrostatic pressure with optimum design using finite element method via ANSYS software. Twenty cases are studied inclusive stiffeners in longitudinal and ring stiffeners. Buckling mode shape is evaluated. This paper studied the optimum design generated by ANSYS for thick cylinder with external hydrostatic pressure. The primary goal of this paper was to identify the improvement in the design of cylindrical shell under hydrostatic pressure with and without Stiffeners (longitudinal and ring) with incorporative technique of an optimization into ANSYS software. The design elements in this research was: critical load, design variable (thickness of shell (TH), stiffener's width (B) and stiffener's height (HF). The results obtained illustrated that the objective is minimized using technique of numerical optimization in ANSYS with optimum shell thickness and stiffener's sizes. In all cases the design variables (thickness of shell) was thicker than the monocoque due to a shell's thicker is essential to achieve the strength constraints. It can be concluded that cases (17,18,19, and 20) have more than 90% of un-stiffened critical load. The ring stiffeners causes increasing buckling load than un-stiffened and longitudinal stiffened cylinder.

KEYWORDS: Buckling, Hydrostatic pressure, stiffeners, cylindrical shell, optimum design, longitudinal stiffeners, ring stiffeners

التصميم الامثل لانبعاج الأسطوانة تحت الضغط الهيدر وستاتيكي الخارجي رواع حامد محمد الجامعة التقنية الوسطى- معهد التكنولوجيا / بغداد- قسم الميكانيك

الخلاصة

يهدف البحث في تقييم الحمل الحرج في الاسطوانة تحت الضغط الهيدروستاتيكي مع التصميم الامثل للاسطوانة من خلال طريقة العناصر المحددة عن طريق برنامج Ansys . تم دراسة عشرون حالة تضمنت تقوية بالاتجاه الطولي واخرى بالاتجاه الحلقي. تم تقييم اطوار الانبعاج لجميع الحالات وتم دراسة التصميم الأمثل للأسطوانة السميكة مع الضغط الهيدروستاتيكي الخارجي. ان الهدف الأساسي من هذا البحث هو تحديد التحسن في تصميم الاسطوانة تحت الضغط الهيدروستاتيكي الخارجي. ان الهدف الأساسي من هذا البحث هو تحديد التحسن في تصميم الاسطوانة تحت الضغط الهيدروستاتيكي الخارجي. ان الهدف الأساسي من هذا البحث هو تحديد التحسن في تصميم الاسطوانة السميكة مع الضغط الهيدروستاتيكي هذا الجارجي. ان الهدف الأساسي من هذا البحث هو تحديد التحسن في تصميم الاسطوانة تحت الضغط الهيدروستاتيكي مع وبدون تقوية (طولية وحلقية) بالاستفادة من تقنية الأمثلية في برنامج Ansys. أما عناصر التصميم في هذا البحث فهي: الحمل الحرج، متغيرات التصميم (سمك القشرة (TH)، عرض المصلب (B) وارتفاع المصلب (HF)، وقد بينت النتائج أن الحمل الحرج تم تقليله عن طريق استخدام التحسين العددي وسمك القشرة المثلى وارتفاع وأحجام المصلب (B) وارتفاع وأحجام التقرية . وفي جميع الأحوال كانت متغيرات التصميم (سمك القشرة (TH)، عرض المصلب (B) وارتفاع وأحجام التورية. وفي جميع الأحوال كانت متغيرات التصميم (سمك القشرة (TH))، عرض المصلب (B) وارتفاع وأحجام التورية . وفي جميع الأحوال كانت متغيرات التصميم (سمك القشرة المتية في برنامج عدي وسمك القشرة المثلى وأحجام التورية أو الحمل الحرج، متغيرات التصميم (سمك القشرة التحسين العددي وسمك القشرة المثلى وأحجام التورية . وفي جميع الأحوال كانت متغيرات التصميم (سمك القشرة) أكبر سمكا من الأحادية لأن القشرة المميكة وأحجام التورية أو وال كانت متغيرات التصميم (سمك القشرة) أكبر سمكا من الأحادية لأن القشرة المتيك وأحجام التورية أو وال كانت من وزول التورية وألمرة الرون القررة) أكبر سمكا من الأحادية وألم وألمو ما وأحجام التورية وأو وال كانت متغيرات التصميم (سمك القشرة) ف وأحجام التقوية أو ول كانت متغيرات التصميم (سمك القشرة) أكبر سمكا من الأحادية لأن القشرة المميكة كانت ضرورية لندي من الرووى المرو والما وألمو والمو واله وألمو الحروم والاسلووانة وما حروم والقوية واللية وألمان (17، 18 ، 19 والي والى من

SYMBOLS

TH	thickness of shell
В	width of stiffener
HF	height of stiffener
DV	design variable
OBJ	objective function
F _{cr}	critical force

INTRODUCTION

The present paper investigates the buckling analysis of cylindrical shell with stiffened and un-stiffened. It had been wide applications in the industry especially in mechanical and nuclear applications, the thick cylinder like submarine pressure hull can buckle through shell instability or lobar buckling of pressure (Ross,2010). Buckling is the general term frequently used to describe the failure structure between the stable and unstable case. When the magnitude of the load on a structure is such that the equilibrium is changed from stable to unstable, the load is called a critical load or (buckling load). When the equilibrium configuration lost its stability, buckling occurs without fracture or material separation (Lyenger and Gupta,1980). Buckling is a problem faced in the structural design, that the members of structural fail under compression loads greater than that material can be resisted. The nature of buckling pattern in plate not only depends upon the type of the applied loading but also on the shape (dimension) of the problem and the material properties and also upon the manner in which the edges are supported (Gerard and Bechler,1957).

A lot of works done to design the cylinder under hydrostatic pressure. The huge challenge is that the buckling strength with critical pressures of the hydrostatic effect. Ross et al., (2004) reported for three ring-stiffened of buckling domes with external hydrostatic pressure. Its contained the theoretical part and experimental part used finite element method. Maalawi (2009) presented a common formulation for the buckling problem with optimization of anisotropic, radially graded, thin-walled, long cylinders subject to external hydrostatic pressure. Hani Aziz Ameen et al., (2012) studied the free vibration of oblate shells i.e. natural frequencies and modes shapes by finite element analysis with and without stiffeners in longitudinal and lateral direction. Numerous parameters were considered like thickness of shell, stiffeners number and stiffeners size. The outcomes were represented by the tenth structural natural frequencies and mode shapes, it would be observed that, the natural frequency of the oblate shell will increase with increasing the mode number and decreased as increasing the shell thickness. Peroti et al., (2013) studied the numerical analysis of ring stiffener effect with various cross section on ultimate buckling strength of pipeline under external hydrostatic pressure. Raju and Rao (2015) studied the multi layered composite shell with reinforced by optimum fiber orientations for pressure vessel also studied the minimization of the mass with strength constraints of cylinder under axial loading under the analysis of static and buckling for the pressure vessel. Qasim and Israa, (2015) studied two types of stiffeners, longitudinal and circumferential stiffeners based on the direction of the installation on the shell surface. The critical pressure due to buckling was calculated numerically by using ANSYS15 for both stiffened and un-stiffened cylinder for various location and installing types. Qasim and Israa, (2016) studied the optimization of critical pressure due to buckling which was calculated numerically by using ANSYS15 for both stiffened and un-stiffened cylinder for various locations and installing types. The optimum design of structure was done by using the ASYS15 program;

The purpose of the current paper is to determine the critical buckling load and mode shapes of the thick cylindrical shell under external hydrostatic pressure with optimum shell design using finite element analysis via ANSYS software program.

FINITE ELEMENT MODEL

The models of thick cylinder with external hydrostatic pressure is done by finite element analysis using the element Shell181. This element is depend on the theory of Reissner / Mindlin thick shell that bending is inclusive also the transverse shear influence. This theory is convenient in modeling the thick cylinder associated with stiffeners. The first stage is analyze the model of cylinder without internal and external stiffeners, with fixed- fixed supporting at its ends. An eigenvalue Buckling analysis will then be done until the solution is converged. The geometry and the mesh model in Ansys are as shown in the Figure(1) (twenty cases are studied). The analysis is carried out for the Steel material of a thick cylinder. Table(1) illustrated the cases studied.

MODE FOR OPTIMIZATION

The outline improvement undertaking is to plan the best size of cylindrical shaped shell presented to external hydrostatic load. The shell can be either solidified with longitudinal or ring hardened by inside and outside rectangular cross-segment rings set at equidistant interims inside the shells. Keeping in mind the end goal to accomplish the ideal plan, the outline advancement process requires a numerical model. This model consists of an objective function, constraints functions, and side constraint functions. These three functions are mathematical equations expressed in terms of design variables and state variables. State variables are fixed quantities in the mathematical equations. This model comprises of an objective function, constraints functions, and side constraint functions. These three capacities are numerical conditions communicated as far as design variables and state variables. State factors are settled amounts in the scientific conditions. Design variables are those amounts which are permitted to change amid the streamlining procedure. The objective function, which inclusive each design variable, is a solitary condition speaking to the amount to be upgraded. So as to demonstrate the above outline assignment, ANSYS programming was utilized to assess the buckling of thick shell under outer hydrostatic pressure, at that point the streamlining issue was assessed by ANSYS programming. ANSYS program figured the buckling loads on either a longitudinal or ring hardened shell, these buckling loads are then come back to the numerical enhancement program, which is joined in the ANSYS programming, to be considered in buckling imperatives on the ideal shell outline. The following steps are the ANSYS APDL for optimization process:

```
/opt
opanl,opt,lgw
opvar,B,dv,0.02,0.1 $ opvar,Hf,dv,0.01,0.1
opvar,Th,dv,0.015,0.1 $ opvar,Fcr1,obj
opvar,Fcr2,obj $ opvar,Fcr3,obj
opvar,Fcr4,obj $ opvar,Fcr5,obj
opvar,Fcr6,obj $ optype,subp
opsubp,15 $ opexe
oplist,all
```

RESULTS AND DISCUSSION

Structural failure by buckling is associated with an unstable design. The ANSYS design the geometry of a stable cylindrical shell based on materials and load factors which are input. Basic disappointment by buckling is related with an unstable design. The buckling happen due to transfer major amount of strain energy into bending energy which leads to sudden failure in the structure. To avoid this type of failure the cylindrical shell must be stiffened by adding stiffeners longitudinal and rings along the shell from internal or external surface. The ANSYS outline the geometry of a stable round and hollow shell in materials light and load factors which are input. In case of cylinder without stiffness, this mode is undesirable, and to improve it, stiffness longitudinal and rings with suitable sized is placed at suitable distances apart in the inside and outside the cylinder's wall. Figure(2) shows the buckling mode shapes (from mode 1 to 6) for all cases which indicates the movement of the cylinder. Table(2) represented the critical loads at each mode shapes and the optimum critical load with optimum sizes of the cylinder (thickness of shell (TH), stiffener's width (B) and stiffener's height (HF) for all cases. It can be observed that cases (1,2 and 3) set 3 and cases (5 and 6) set 4 and cases (7 and 8) set 3 and so on the other cases, its similar in sizes and different in the critical load. The outcomes for the longitudinal and ring-solidified shell are exhibited in an indistinguishable configuration from the monocoque shell; objective, plan factors and requirements. There are three outline of factors rather than one. This outcomes is a more mind boggling enhancement issue. Table(3) illustrated the ratio of the incremental critical load with stiffeners and its observed that cases (17,18,19, and 20) have more than 90% of un-stiffened critical force. In particular, the tallness and width of the stiffeners changed extensively. The benefit of utilizing enhancement in ANSYS programming for shell configuration was acknowledged by an examination of the imperatives. By and large ANSYS fulfilled the buckling imperatives for both monocoque and longitudinal and ring hardened shell. However a huge contrast was found in sizes and basic burdens.

CONCLUSIONS

The main aim of this research is to identify the improvement of the design of cylindrical shell under external hydrostatic pressure with and without Stiffeners (longitudinal and ring) since the incorporative of the technique optimization into ANSYS software. The design elements in this research was: objective (critical load), design variable (thickness of shell (TH), stiffener's width (B) and stiffener's height (HF). The results shown the minimization of the objective by the technique of numerical optimization in ANSYS with optimum shell thickness and stiffener's sizes. In all cases the design variables (thickness of shell) was thicker than the monocoque since a shell's thicker is essential to achieve the strength constraints and it can be concluded that cases (17,18,19, and 20) have more than 90% of un-stiffened critical load. The ring stiffeners causes increasing buckling load than un stiffened and longitudinal stiffened cylinder .

Table(1) Cases studied

cases	Description
1	cylindrical shell without stiffeners
2	cylindrical shell with two longitudinal stiffeners inward
3	cylindrical shell with four longitudinal stiffeners inward
4	cylindrical shell with two longitudinal stiffeners outward
5	cylindrical shell with four longitudinal stiffeners outward
6	cylindrical shell with four longitudinal stiffeners inward and outward
7	cylindrical shell with two ring stiffeners inward
8	cylindrical shell with four ring stiffeners inward
9	cylindrical shell with six ring stiffeners inward
10	cylindrical shell with eight ring stiffeners inward
11	cylindrical shell with ten ring stiffeners inward
12	cylindrical shell with two ring stiffeners outward
13	cylindrical shell with four ring stiffeners outward
14	cylindrical shell with six ring stiffeners outward
15	cylindrical shell with eight ring stiffeners outward
16	cylindrical shell with ten ring stiffeners outward
17	cylindrical shell with ten ring stiffeners inward and outward
18	cylindrical shell with four longitudinal stiffeners inward and ten ring stiffeners inward
19	cylindrical shell with four longitudinal stiffeners outward and ten ring stiffeners outward
20	cylindrical shell with four longitudinal stiffeners inward and outward and ten ring stiffeners inward and outward



Figure.1 Cases studied , un-stiffened and stiffened cylinder

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case-1-	O mode-1-	O mode-2-	O mode-3-	mode-4-	mode-5-	mode-6-
case-2-	O	O	O mode-3-	O mode-4-	O moda-5-	D mode-6-
case-3-	O	O mode.2	O mode-3-	O mode-4-	D mode-5-	D mode-6-
case-4-	Mode-1-	Mode-2-	O mode-3-	O mode-4-	mode-5-	Mode-6-
case-5-	mode-1-	O mode-2-	O mode-3-	Mode-4-	mode-5-	mode-6-
case-6-	O mode-1-	Mode-2-	O mode-3-	mode-4-	D mode-5-	D mode-6-
case-7-	mode-1-	mode-2-	Mode-3-	D mode-4-	D mode-5-	Mode-6-
case-8-	mode-1-	mode-2-	Mode-3-	mode-4-	mode-5-	D mode-6-
case-9-	mode-1-	mode-2-	O mode-3-	Mode-4-	mode-5-	mode-6-
case-10-	Mode-1-	Mode-2-	Mode-3-	Mode-4-	Mode-5-	O mode-6-



Figure.2. Mode shapes of cylindrical stiffener shell under hydrostatic pressure

cases		F _{cr} (N)		Optimization parameters					
Case	Б	2597242		(a	* symbol is u	sed to indicate	the best listed	set)	
-1-	Γ _{CR}	2387343			*SEI 1*	SEI 2	SEI 3	SEI 4	
	I F _{CR}	2667954			E	FEASIBL E	E	FEASIBLE	
	2	4	ТН	DV	1 50E-02	8 70E-02	5 39E-02	1 54E-02	
	F _{CR}	2705554	FCR	OBI	2.59E+07	2 59E+07	2.59E+07	2.59E+07	
	3	2	I CK	0.00	2.371107	2.371107	2.371107	2.3711107	
	F _{CR}	2726918							
	4 F	5950427							
	Γ_{CR}	5859427 1							
	F _{CR}	5930985							
	6	8							
Case	F _{CR}	2629579			SET 1	SET 2	*SET 3*	SET 4	
-2-	1	5			FEASIBL	FEASIBL	FEASIBL	FEASIBL	
	F _{CR}	2/28861		DU	E	E	E	E	
	2 Ecp	3004022	B	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02	
	1 CK	5	HF		1.00E-02	5.11E-02	5.84E-02	1.03E-02	
	F _{CR}	3286792	TH		1.50E-02	4.80E-02	6.75E-02	5.49E-02	
	4	5	F _{CR}	OBJ	2.63E+07	2.66E+07	2.66E+07	2.64E+07	
	F _{CR}	5916692							
	5	3							
	F _{CR}	5963926							
Case	6	2637812			CET 1	CET 2	*CET 2*	CET 4	
-3-	E _{CP1}	2037812			SET T FEASIBI	SEI 2 FEASIBI	*SE1 3*	SEI 4 EEASIBI	
	I CRI	2732007			E	FLASIBL	E	E	
	F _{CR2}	1	B	DV	2.00E-02	8 78E-02	7 54E-02	8 78E-02	
	0112	3010909	HF	DV	1.00E-02	5.11E-02	5 84E-02	1.03E-02	
	F _{CR3}	2	TH	DV	1.50E-02	4 80E-02	6.75E-02	5.49E-02	
		3301651	FCP	OBJ	2.64E+07	2.68E+07	2.68E+07	2.65E+07	
	F _{CR4}	6	I CK	0.50	2.012107	2.002107	2.002107	21002107	
	Б	5922083							
	Γ _{CR5}	597/797							
	FCP6	8							
Case	FCR1	25911800			SET 1	SET 2	*SET 3*	SET 4	
-4-	FCR2	26766474			FEASIBL	FEASIBL	FEASIBL	FEASIBL	
	F _{CR3}	27074520			Е	E	Е	E	
	F _{CR4}	27314982	В	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02	
	F _{CR5}	58690389	HF	DV	1.00E-02	5.11E-02	5.84E-02	1.03E-02	
	F _{CR6}	59349511	TH	DV	1.50E-02	4.80E-02	6.75E-02	5.49E-02	
		·	F _{CR}	OBJ	2.59E+07	2.60E+07	2.60E+07	2.60E+07	
Case	F _{CR1}	25980569			SET 1	SET 2	SET 3	*SET 4*	

Table(2) Critical Load and Optimization parameters to all cases study

	Г	0(0)75(0			EEACIDI	EEACIDI	EEACIDI	EEACIDI
	F _{CR2}	2683/568			FEASIDL E	FEASIDL E	FEASIDL E	FEASIDL
	F _{CR3}	27190803	B	DV	2 00E-02	E 8 78E_02	E 7 54E-02	E 8 78E-02
	F _{CR4}	2/504341	Б ЦЕ		1.00E-02	5.11E.02	7.34E-02	1.03E.02
	F _{CR5}	58788901	тц		1.00E-02	4 80E 02	5.84E-02	1.03E-02
	F _{CR6}	59477169	E.	OPI	1.50E-02	4.80E-02	0.75E-02	$2.61E \pm 07$
Case	Б	26162721	TCR	ODJ	2.00L+07	2.02L+07	2.02L+07	2.01L+07
-6-	Γ_{CR1}	20402734			SET T	SET 2 FEASIBI	SET S FEASIBI	FEASIBI
_	F _{CR2}	27409082			F	F	F	F
	F _{CR3}	30253443	R	DV	2 00F-02	8 78F-02	7 54E-02	8 78E-02
	F _{CR4}	33296495	HE	DV	1.00E-02	5.11E-02	7.34E-02	1.03E-02
	F _{CR5}	59337270			1.00E-02	1 80E 02	5.84E-02	1.03E-02
	F _{CR6}	59873355	Б		1.30E-02	4.60E-02	0.73E-02	3.49E-02
			Γ_{CR}	OBJ	2.03E+07	2.72E+07	2.72E+07	2.0/E+0/
Case	F _{CR1}	28980332			SET 1	SET 2	*SET 3*	SET 4
-7-	F _{CR2}	30057685				FEASIBL	FEASIBL	FEASIBL
	F _{CR3}	32197776			FEASIBLE	E	E	E
	F _{CR4}	32571040	В	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02
	F _{CR5}	63866892	HF	DV	1.00E-02	5.11E-02	5.84E-02	1.03E-02
	F _{CR6}	64172633	TH	DV	1.50E-02	4.80E-02	6.75E-02	5.49E-02
			F _{CR}	OBJ	2.90E+07	7.47E+07	7.48E+07	6.56E+07
Case	F _{CR1}	31520435			SET 1	SET 2	*SET 3*	SET 4
-8-	F _{CR2}	32274272				FEASIBL	FEASIBL	FEASIBL
	F _{CR3}	34740105			FEASIBLE	E	E	E
	F _{CR4}	34816924	В	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02
	F _{CR5}	67973609	HF	DV	1.00E-02	5.11E-02	5.84E-02	1.03E-02
	F _{CR6}	68256103	TH	DV	1.50E-02	4.80E-02	6.75E-02	5.49E-02
0			F _{CR}	OBJ	3.15E+07	9.95E+07	9.89E+07	8.73E+07
Lase	F _{CR1}	33927193			SET 1	SET 2	SET 3	*SET 4*
,	F_{CR2}	34569617				FEASIBL	FEASIBL	FEASIBL
	F _{CR3}	35970591	D	DV	TEASIBLE	0 70E 00		E 0 70E 00
	F _{CR4}	36018867	D		2.00E-02	δ./δΕ-02	7.54E-02	δ./δE-02
	F _{CR5}	73058288			1.00E-02	J.11E-02	J.84E-02	1.05E-02
	F _{CR6}	73689073			1.50E-02	4.80E-02	0.75E-02	J.49E-02
Casa	Г	26020502	г _{CR}	ORI	3.39E+0/	1.48E+08	1.48E+08	1.28E+U8
-10-	F _{CR1}	36238593			SETI	SET 2	SET 3	[↑] SET 4*
	F _{CR2}	36423677			FEASIRIE	F	F	F
	F _{CR3}	364/91/7	р	DV		0 70E 07	754E02	
	F _{CR4}	36/9148/	л		1 ODE-02	5 11E 02	5 8/E 02	1 03E 02
	F _{CR5}	78309589	ти ти		1.00E-02	/ 20E 02	675E 02	5 /0E 02
			111	יע	1.JUE-02	4.00E-02	0.73E-02	J.47E-02

	F _{CR6}	79292419	F _{CR}	OBJ	3.62E+07	2.84E+08	2.83E+08	2.23E+08
Case	F _{CR1}	36563142			SET 1	SET 2	*SET 3*	SET 4
-11-	F _{CR2}	36589965				FEASIBL	FEASIBL	FEASIBL
	F _{CR3}	38270359			FEASIBLE	Е	E	E
	F _{CR4}	38588594	В	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02
	F _{CR5}	80209568	HF	DV	1.00E-02	5.11E-02	5.84E-02	1.03E-02
	F _{CR6}	80819308	TH	DV	1.50E-02	4.80E-02	6.75E-02	5.49E-02
	0110		F _{CR}	OBJ	3.66E+07	3.48E+08	3.47E+08	2.08E+08
Case	F _{CR1}	28809314			SET 1	SET 2	*SET 3*	SET 4
-12-	F _{CR2}	29806790				FEASIBL	FEASIBL	FEASIBL
	F _{CR3}	31850636			FEASIBLE	E	E	E
	F _{CR4}	32038528	В	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02
	F _{CR5}	63509754	HF	DV	1.00E-02	5.11E-02	5.84E-02	1.03E-02
	F _{CR6}	63855711	TH	DV	1.50E-02	4.80E-02	6.75E-02	5.49E-02
			F _{CR}	OBJ	2.88E+07	7.65E+07	7.69E+07	6.63E+07
Case	F _{CR1}	31094758			SET 1	SET 2	*SET 3*	SET 4
-13-	F _{CR2}	31822533				FEASIBL	FEASIBL	FEASIBL
	F _{CR3}	34128701			FEASIBLE	E	E	E
	F _{CR4}	34211167	В	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02
	F _{CR5}	67240763	HF	DV	1.00E-02	5.11E-02	5.84E-02	1.03E-02
	F _{CR6}	67475483	TH	DV	1.50E-02	4.80E-02	6.75E-02	5.49E-02
			F _{CR}	OBJ	3.11E+07	9.91E+07	9.85E+07	8.55E+07
Case	F _{CP1}	33516038			SET 1	SET 2	*SET_3*	SET 4
-14-	FCR2	34056782				FEASIBL	FEASIBL	FEASIBL
	F _{CP2}	35357072			FEASIBLE	Е	Е	Е
	F _{CP4}	35462807	В	DV	2.00E-02	8.78E-02	7.54E-02	878E_02
	F _{CP5}	72345444	HF	DV	1.00E-02		5 94E 02	0.701-02
	FCR6	72481891	тц		1.001 02	5.11E-02	J.04E-02	1.03E-02
	- CK0	12 + 01071	п	DV	1.50E-02	5.11E-02 4.80E-02	6.75E-02	1.03E-02 5.49E-02
Case	i	72401071	F _{CR}	DV OBJ	1.50E-02 3.35E+07	5.11E-02 4.80E-02 1.49E+08	6.75E-02 1.49E+08	1.03E-02 5.49E-02 1.22E+08
-15-	F _{CR1}	36088468	F _{CR}	DV OBJ	1.50E-02 3.35E+07 SET 1	5.11E-02 4.80E-02 1.49E+08 SET 2	6.75E-02 1.49E+08 SET 3	1.03E-02 5.49E-02 1.22E+08 *SET 4*
	F _{CR1} F _{CR2}	36088468 36461187	F _{CR}	DV OBJ	1.50E-02 3.35E+07 SET 1	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL	5.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL	1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL
	F _{CR1} F _{CR2} F _{CR3}	36088468 36461187 39712098	F _{CR}	DV OBJ	1.50E-02 3.35E+07 SET 1 FEASIBLE	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL E	5.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL E	1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL E
	F_{CR1} F_{CR2} F_{CR3} F_{CR4}	36088468 36461187 39712098 39916032	F _{CR}	DV OBJ DV	1.50E-02 3.35E+07 SET 1 FEASIBLE 2.00E-02	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL E 8.78E-02	5.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL E 7.54E-02	1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL E 8.78E-02
	F_{CR1} F_{CR2} F_{CR3} F_{CR4} F_{CR5}	36088468 36461187 39712098 39916032 78917833	In F _{CR} B HF	DV OBJ DV DV	1.50E-02 3.35E+07 SET 1 FEASIBLE 2.00E-02 1.00E-02	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL E 8.78E-02 5.11E-02	5.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL E 7.54E-02 5.84E-02	1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL E 8.78E-02 1.03E-02
	F_{CR1} F_{CR2} F_{CR3} F_{CR4} F_{CR5} F_{CR6}	36088468 36461187 39712098 39916032 78917833 79491842	IH F _{CR} B HF TH	DV OBJ DV DV DV	1.50E-02 3.35E+07 SET 1 FEASIBLE 2.00E-02 1.00E-02 1.50E-02	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL E 8.78E-02 5.11E-02 4.80E-02	3.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL E 7.54E-02 5.84E-02 6.75E-02	1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL E 8.78E-02 1.03E-02 5.49E-02
	$\begin{array}{c} F_{CR1} \\ F_{CR2} \\ F_{CR3} \\ F_{CR4} \\ F_{CR5} \\ F_{CR6} \end{array}$	36088468 36461187 39712098 39916032 78917833 79491842	IH F _{CR} B HF TH F _{CR}	DV OBJ DV DV DV OBJ	1.50E-02 3.35E+07 SET 1 FEASIBLE 2.00E-02 1.50E-02 3.61E+07	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL E 8.78E-02 5.11E-02 4.80E-02 2.89E+08	3.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL E 7.54E-02 5.84E-02 6.75E-02 2.88E+08	1.03E-02 1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL E 8.78E-02 1.03E-02 5.49E-02 2.07E+08
Case	F_{CR1} F_{CR2} F_{CR3} F_{CR4} F_{CR5} F_{CR6}	36088468 36461187 39712098 39916032 78917833 79491842 35962903	IH F _{CR} B HF TH F _{CR}	DV OBJ DV DV DV OBJ	1.50E-02 3.35E+07 SET 1 FEASIBLE 2.00E-02 1.50E-02 1.50E-02 3.61E+07 SET 1	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL E 8.78E-02 5.11E-02 4.80E-02 2.89E+08 SET 2	5.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL E 7.54E-02 5.84E-02 6.75E-02 2.88E+08 SET 3	1.03E-02 1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL E 8.78E-02 1.03E-02 5.49E-02 2.07E+08 *SET 4*
Case -16-	$\begin{array}{c} F_{CR1} \\ F_{CR2} \\ F_{CR3} \\ F_{CR4} \\ F_{CR5} \\ F_{CR6} \\ \end{array}$	36088468 36461187 39712098 39916032 78917833 79491842 35962903 36130090	IH F _{CR} B HF TH F _{CR}	DV OBJ DV DV DV OBJ	1.50E-02 3.35E+07 SET 1 FEASIBLE 2.00E-02 1.50E-02 3.61E+07 SET 1	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL E 8.78E-02 5.11E-02 4.80E-02 2.89E+08 SET 2 FEASIBL	3.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL E 7.54E-02 5.84E-02 6.75E-02 2.88E+08 SET 3 FEASIBL	1.03E-02 1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL E 8.78E-02 1.03E-02 5.49E-02 2.07E+08 *SET 4* FEASIBL FEASIBL
Case -16-	F_{CR1} F_{CR2} F_{CR3} F_{CR4} F_{CR5} F_{CR6} F_{CR1} F_{CR2} F_{CR2}	36088468 36461187 39712098 39916032 78917833 79491842 35962903 36130090 39019898	IH F _{CR} B HF TH F _{CR}	DV OBJ DV DV DV OBJ	1.50E-02 3.35E+07 SET 1 FEASIBLE 2.00E-02 1.50E-02 3.61E+07 SET 1	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL E 8.78E-02 5.11E-02 4.80E-02 2.89E+08 SET 2 FEASIBL E	3.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL E 7.54E-02 6.75E-02 2.88E+08 SET 3 FEASIBL E	1.03E-02 1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL E 8.78E-02 1.03E-02 5.49E-02 2.07E+08 *SET 4* FEASIBL E *SET 4* FEASIBL E
Case -16-	$\begin{array}{c} F_{CR1} \\ F_{CR2} \\ F_{CR3} \\ F_{CR4} \\ F_{CR5} \\ F_{CR6} \\ \end{array}$	36088468 36461187 39712098 39916032 78917833 79491842 35962903 36130090 39019898 39237880	IH F _{CR} B HF TH F _{CR} B	DV OBJ DV DV OBJ DV	1.50E-02 3.35E+07 SET 1 FEASIBLE 2.00E-02 1.50E-02 3.61E+07 SET 1 FEASIBLE 2.00E-02 1.50E-02 3.61E+07 SET 1 FEASIBLE 2.00E-02	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL E 8.78E-02 5.11E-02 4.80E-02 2.89E+08 SET 2 FEASIBL E 8.78E-02	3.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL E 7.54E-02 6.75E-02 2.88E+08 SET 3 FEASIBL E 7.54E-02 6.75E-02 2.88E+08 SET 3 FEASIBL E 7.54E-02	1.03E-02 1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL E 8.78E-02 1.03E-02 5.49E-02 2.07E+08 *SET 4* FEASIBL E 8.78E-02 2.07E+08 *SET 4* FEASIBL E 8.78E-02 8.78E-02
Case -16-	$\begin{array}{c} F_{CR1} \\ F_{CR2} \\ F_{CR3} \\ F_{CR4} \\ F_{CR5} \\ F_{CR6} \\ \end{array}$	36088468 36461187 39712098 39916032 78917833 79491842 35962903 36130090 39019898 39237880 79320063	IH F _{CR} B HF TH F _{CR} B HF	DV OBJ DV DV OBJ OV DV OBJ	1.50E-02 3.35E+07 SET 1 FEASIBLE 2.00E-02 1.50E-02 3.61E+07 SET 1 FEASIBLE 2.00E-02 1.50E-02 3.61E+07 SET 1 FEASIBLE 2.00E-02 1.00E-02 1.00E-02	5.11E-02 4.80E-02 1.49E+08 SET 2 FEASIBL E 8.78E-02 5.11E-02 4.80E-02 2.89E+08 SET 2 FEASIBL E 8.78E-02 5.11E-02	3.84E-02 6.75E-02 1.49E+08 SET 3 FEASIBL E 7.54E-02 6.75E-02 2.88E+08 SET 3 FEASIBL E 7.54E-02 5.84E-02 5.84E-02 5.84E-02 5.84E-02 5.84E-02 5.84E-02 5.84E-02 5.84E-02	1.03E-02 1.03E-02 5.49E-02 1.22E+08 *SET 4* FEASIBL E 8.78E-02 1.03E-02 5.49E-02 2.07E+08 *SET 4* FEASIBL E 8.78E-02 1.03E-02 1.03E-02 1.03E-02 1.03E-02

			F _{CR}	OBJ	3.60E+07	3.64E+08	3.63E+08	1.93E+08
Case	F _{CR1}	43372167			SET 1	SET 2	*SET 3*	SET 4
-17-	F _{CR2}	43474208				FEASIBL	FEASIBL	FEASIBL
	F _{CR3}	48014965			FEASIBLE	Е	Е	Е
	F _{CR4}	48253288	В	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02
	F _{CR5}	94897468	HF	DV	1.00E-02	5.11E-02	5.84E-02	1.03E-02
	F _{CR6}	95640820	TH	DV	1.50E-02	4.80E-02	6.75E-02	5.49E-02
	CRU		F _{CR}	OBJ	4.34E+07	3.87E+08	3.88E+08	2.93E+08
Case	F _{CR1}	37800633			SET 1	SET 2	*SET 3*	SET 4
-18-	F _{CR2}	38012255				FEASIBL	FEASIBL	FEASIBL
	F _{CR3}	40989265			FEASIBLE	Е	Е	Е
	F _{CR4}	43351175	В	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02
	F _{CR5}	81170628	HF	DV	1.00E-02	5.11E-02	5.84E-02	1.03E-02
	F _{CR6}	81971530	TH	DV	1.50E-02	4.80E-02	6.75E-02	5.49E-02
			F _{CR}	OBJ	3.78E+07	4.55E+08	4.51E+08	2.13E+08
Case	F _{CR1}	36268408			SET 1	SET 2	*SET 3*	SET 4
-19-	F _{CR2}	36414993				FEASIBL	FEASIBL	FEASIBL
	F _{CR3}	39150188			FEASIBLE	E	E	E
	F _{CR4}	39272996	В	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02
	F _{CR5}	79740460	HF	DV	1.00E-02	5.11E-02	5.84E-02	1.03E-02
	F _{CR6}	80588314	TH	DV	1.50E-02	4.80E-02	6.75E-02	5.49E-02
			F _{CR}	OBJ	3.63E+07	4.66E+08	4.63E+08	1.96E+08
Case	F _{CR1}	45594726			SET 1	SET 2	SET 3	*SET 4*
-20-	F _{CR2}	46030296				FEASIBL	FEASIBL	FEASIBL
	F _{CR3}	50272434			FEASIBLE	E	E	E
	F _{CR4}	52154951	В	DV	2.00E-02	8.78E-02	7.54E-02	8.78E-02
	F _{CR5}	96644758	HF	DV	1.00E-02	5.11E-02	5.84E-02	1.03E-02
	F _{CR6}	97838532	TH	DV	1.50E-02	4.80E-02	6.75E-02	5.49E-02
			FC					
			R	OBJ	4.56E+07	5.27E+08	5.26E+08	3.36E+08

	$T_{1} = r_{1} + \frac{1}{2} $
cases	The ratio of increment %
2	2.631578947
3	3.358208955
4	0.384615385
5	0.766283525
6	2.996254682
7	65.37433155
8	73.81193124
9	79.765625
10	88.38565022
11	92.53602305
12	66.31989597
13	73.70558376
14	82.61744966
15	87.48792271
16	86.58031088
17	93.32474227
18	94.25720621
19	94.40604752
20	92.29166667

Table(3) The ratio of the incremental critical load with stiffeners

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