The Iraqi Journal for Mechanical and Material Engineering, Vol.23, No1,Jun.2024

AN EXTENSIVE REVIEW OF VIBRATION METHODS FOR BONE HEALING ASSESSMENT

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ABSTRACT

The present study offers a thorough examination of the utilization of vibration techniques in the evaluation of bone healing in the domain of orthopedics and rehabilitation. In addition to providing critical insights into the mechanical properties of healing bones and their influence on treatment choices, the review includes an investigation of objective, non-invasive techniques for tracking the advancement of bone healing. The study also explores the function of vibration-based assessment methods in treatment planning and how they support ongoing research and development initiatives. Additionally, the development of this important field is clarified by discussing the historical background and current bone mechanics research initiatives. The goal of the review is to present a comprehensive understanding of vibration methods for assessing bone healing and their possible applications in clinical practice and research. The outcomes of this review state that the non-invasive techniques help clinicians make well-informed treatment decisions by offering insightful information about bone mechanics. In addition, the review highlights the potential influence of vibration methods on clinical practice and research, indicating potential for improving orthopedic care.

keywords:vibration,bone healing

INTRODUCTION

Vibration techniques have become an important tool in orthopedics and rehabilitation for determining bone healing. These methods provide objective, non-invasive ways to evaluate and track the development of bone healing, giving important information about the process and supporting treatment choices. These methods can be used to improve the monitoring and assessment of recovery. Vibration-based approaches provide a quantitative assessment that is simple to repeat over time, in contrast to more invasive and subjective traditional methods like X-rays and manual palpation. This makes it possible to precisely monitor the bone-healing process, which makes it easier to identify healing early and take appropriate action when needed. Secondly, vibration techniques provide important information about the rate and quality of fracture healing by evaluating the mechanical properties of the healing bone. It is possible to obtain quantitative information about the mechanical behavior of the healing bone through the analysis of parameters like resonance frequency, damping coefficient, or wave propagation properties. This makes it easier to track the healing process and gives medical practitioners the information they need to choose the best course of action. Furthermore, the process of planning a treatment plan for bone healing heavily relies on vibration-based assessment techniques. Clinicians can decide when weight-bearing, physical therapy, or surgery is the best course of action by objectively measuring the healing process. By creating personalized plans for each patient, healthcare professionals can maximize treatment and enhance results. Lastly, a contribution to the ongoing research and development efforts is the use of vibration techniques in the detection of bone healing. By improving our understanding of the processes that underlie bone healing and how these connect to vibrational characteristics, researchers can advance the field and look into new treatment approaches. This data may stimulate the development of new treatments or strategies to enhance bone healing outcomes. In order to understand the goal of the current work, a thorough literature review will be presented in the following sections. This will involve examining the work and conclusions of others.

HISTORICAL BACKGROUND

The study of bone strength was an important area of research to evaluate both normal and abnormal physiological conditions. Developing a precise diagnostic method for accurately assessing the health of the bone is the aim of these studies. Galileo's research was groundbreaking in the field of bone mechanics, which has been studied since [G. Galileo, 1638].

The first study on femur mechanics was published by [John C. Koch, 1917] using common beam theories. This study illustrated how the stress contours are influenced by the geometric shape of the femur. This geometric configuration increased the maximum strength to femur mass ratio. The primary goal of those earlier studies was to calculate bone strength under static loading conditions. The industry standard for diagnosis and prediction was established in the last century with the development of two devices: QCT and DEXA. These two methods measured the x-ray beam's attenuation in the medium to determine the bone mineral density (BMD). Both approaches are very accurate, but they have drawbacks like radiation exposure, costly equipment, and statistical analysis of bone characteristics. Several studies have shown that fractures can still happen in bones even in cases where bone mineral density as determined by DEXA and QCT is within normal limits [Susan M., Ott, 1993]. Because the response of the bone to dynamic loading is very different from that to static loading, this discovery led to the realization that the structural dynamic properties of the bone need to be evaluated. The investigation of structural dynamic properties used two methods. In the first case, the natural frequency change was studied; in the second, the bending stiffness was studied. The wave propagation technique was employed in both approaches to analyze frequencies. Numerous analytical and numerical methods, such as modal analysis, transfer matrix analysis, and finite element analysis, were used to evaluate these two parameters. The experimental evaluation was essentially constructed using the impulse test technique. With this technique, the structure is excited using an impulse hammer or shaker. The natural frequency, or bending stiffness, is ascertained by utilizing both the excitation and the response. These investigations were carried out both in vitro and in vivo. The objective of these investigations was to create an in-vivo testing apparatus that could precisely assess the structural and dynamic characteristics of bone tissue. The studies that were previously carried out to assess bone integrity using frequency analysis are shown in the following section.

There are several reasons why frequency response techniques are important in addition to that. Its primary significance comes from its ability to provide important new understandings of the mechanical characteristics of healing bone tissue. This method allows the measurement of bone stiffness, damping properties, and structural integrity by examining the bone's response to vibrations at various frequencies. Due to its ability to detect minute variations in the characteristics of the bone that would be missed by more conventional methods of assessment, this method is essential for tracking the process of bone healing. It gives medical professionals a quantitative, unbiased way to track the healing process and track variations in bone strength and structural stability over time. The non-invasive nature of the frequency response method adds to its suitability for application in clinical settings. Without requiring invasive procedures or exposure to ionizing radiation, clinicians can use controlled vibrations to gather important data on the response of the bone.

However, the frequency response approach can also help with treatment selection. Medical practitioners can assess when weight-bearing, physical therapy, or surgical procedures are most suitable by tracking changes in bone properties over time. Tailored treatment improves outcomes and patient care. The frequency response method also advances research and development in the field of bone healing. By analyzing the relationship between vibrational properties and bone healing results, researchers can gain more insight into the underlying mechanisms and potentially develop novel therapeutic approaches to enhance bone healing.

TRADITIONAL METHODS FOR MEASURING STRENGTH AND THEIR LIMITATIONS

Bone strength diagnostic techniques are largely used to assess the integrity and health of the bones. These methods are essential for determining abnormalities, evaluating bone strength, and determining the best course of action. This section presents the many types of diagnostic techniques as well as the general significance of bone strength assessment. Furthermore, there are three categories in which the significance of bone strength diagnostic techniques can be categorized:

Fracture Risk Assessment

The likelihood of fractures can be determined using diagnostic methods based on bone strength. By assessing bone strength or density, these tests offer crucial information regarding bone health and fracture risk, particularly in conditions like osteoporosis or osteopenia.

Treatment Planning

A precise evaluation of bone strength helps with treatment planning. Diagnostic methods help determine whether therapies like medication therapy, surgery, or lifestyle modifications are necessary to improve bone health and reduce the risk of fracture.

Monitoring Bone Health

Diagnostic testing can be used to monitor bone health over time. They encourage tracking alterations in bone mass or density and assess the efficacy of interventions or therapies aimed at maintaining or improving bone health. In general, there are several types of bone strength diagnostic techniques, some of them are described below as:

DEXA, or Dual-Energy X-ray Absorptiometry

Is a frequently used method (illustrated in Fig. 1.2) for measuring bone mineral density (BMD). It provides information on the bone density at specific skeletal locations, such as the hip, spine, and forearm. DEXA is widely used to assess fracture risk, identify osteoporosis, and monitor treatment efficacy. To attenuate the x-ray inside the bone, this imaging device uses two x-ray beams: one for high energy and one for low energy. The amplitude of the attenuation is directly related to the bone mineral content (BMC). The ratio of the projected area to the bone mineral content is used to compute bone mineral density [C. C. Djokoto, et al, 2002].

Bone mineral density (BMD) and bone health are evaluated using the widely used Dual-Energy X-ray Absorptiometry (DEXA) technique. It is expensive and difficult to use, and it has drawbacks like a restricted evaluation of bone quality, site-specific measurement, incapacity to distinguish between cortical and trabecular bone, radiation exposure, and interference from body composition. Furthermore, the cost of acquisition and upkeep of DEXA may restrict its use in specific healthcare environments.

Quantitative Computed Tomography (QCT)

QCT uses computed tomography imaging technology to measure bone strength and mineral density. It provides a three-dimensional assessment of bone quality that allows differentiation between cortical and trabecular bone. When evaluating the strength of the bones in the hips and spine, QCT is particularly useful. Fig. 1.3 depicts a QCT gadget.

Among the disadvantages are radiation exposure, equipment needs, increased expense, restricted soft tissue differentiation, image artifacts, and constrained clinical guidelines. QCT usually requires higher radiation doses than Dual-Energy X-ray Absorptiometry (DEXA), which raises concerns for long-term studies or frequent monitoring. In addition, QCT necessitates specific CT scanners, which might not be easily accessible in some medical environments. Additionally, the increased expense prevents QCT from being widely used as a standard diagnostic tool.

Quantitative Ultrasound (QUS)

QUS methods employ high-frequency sound waves to assess the density and strength of bones. These tests can provide information about the composition and quality of bone by measuring the transmission and reflection of sound waves through bone. QUS is displayed in Fig. 1.4 for assessing the state of the bones in peripheral skeletal sites such as the heel or finger.

It may not offer thorough information about central skeletal sites like the spine or hip because it is site-specific and mainly used for peripheral skeletal sites. Reproducibility can be limited by operator dependence, which can impact the precision and quality of QUS measurements. QUS results can also be impacted by soft tissues' influence and a lack of standardization. QUS may not provide thorough insights into other aspects of bone quality, but it does provide information about bone density and stiffness. The precision and dependability of QUS measurements can be impacted by patient characteristics such as obesity, edema, or skeletal abnormalities.

DYNAMIC ANALYSIS AND FREQUANCY RESPONSE METHOD

Dynamic analysis of bones is the study of how bones behave mechanically under dynamic loading scenarios like impacts and vibrations. Understanding bone biomechanics, assessing fracture risk, and developing effective medical interventions all depend on understanding bone kinetics. The frequency analysis method is one of the most crucial techniques for the dynamic analysis. In order to precisely measure bones' mechanical properties under dynamic loading, frequency analysis of bones was first conducted in the 1950s. Because it provides important information about bone stiffness, natural frequencies, and damping properties, frequency response analysis is essential to bone testing. A technique used to find out how a system, like a bone, responds to various applied force or vibration frequencies is called frequency response analysis. In the context of bone testing, this means measuring the bone's response to intentional vibrations or mechanical stimulation at different frequencies. Bone displacement and acceleration are frequently used to describe the measurement of this response.

The ability of frequency response analysis to reveal the basic mechanical characteristics and behaviors of bones makes it important for bone testing. Numerous factors make frequency response analysis useful, including:

Analysis of Frequency Response**:** helps determine how flexible and rigid bones are. By observing how the bone reacts to known forces or vibrations at different frequencies, one can infer certain characteristics of the bone's stiffness. Comprehending this data is essential for appraising load-bearing ability, determining bone health, and appreciating bone strength.

Frequencies Found in Nature: Every bone has its own inherent frequencies at which it prefers to oscillate. The frequencies at which the bone exhibits the highest degree of response can be found by subjecting it to a range of frequencies. In addition to providing useful information about a bone's resonant patterns and structural features, natural frequencies of bones can be used to diagnose bone disorders or identify abnormalities.

Features of Damping**:** The damping qualities of bones can also be assessed with the help of frequency response analysis. A substance's ability to release energy during vibration is known as damping. By examining the bone's reaction and contrasting it with a known excitation force, one can estimate the bone's damping characteristics. Understanding the damping characteristics is crucial for estimating the healing time of fractures and evaluating a bone's capacity to withstand trauma.

Frequency response analysis is generally essential for dynamic analysis and bone testing. It helps to understand bone biomechanics and provides useful information on bone stiffness, natural frequencies, and damping properties. By utilizing frequency response analysis techniques, scientists and medical professionals can improve their understanding of bone dynamics, improve fracture risk assessment, and develop more effective treatments for bone-related conditions.

Vibration Analysis using Finite Element Method

Timothy K. Hight [7] examined the tibia bone because of its unique shape in relation to other long bones and its middle region's thin soft tissue. The author of the study calculated the tibia natural frequency of the bone using a finite element model, taking into consideration the effects of different boundary conditions at the proximal and distal ends as well as the effects of shape irregularity. The results showed that the accuracy differences between the lumped mass and continuous systems were negligible. This difference was also crucial in distinguishing the accuracy of the two approaches. Both the bone's rotating inertia and center of curvature had an equal impact on its weight. The natural frequency also changed in response to a modification in the boundary condition. By using three dimensional finite element methods to create a three-dimensional finite element model of a human tibia, Hobatho, M. C. et al. (1988) used more complex analysis. The objective of the finite element analysis was to determine the natural frequency tibia. The approximations made in the length and inertia of the previous studies were eliminated by this analysis. Even with such a model, there was a mismatch between the experimental results and the theoretical analysis. The distribution of mass and mechanical properties within the bone had an impact on this disparity. The accuracy of the finite element analysis increased when the mechanical properties and mass distribution of the bone were considered [Hobatho, M. C, et al, 1988]. A study on human tibia modeling was presented by [Thomsen, J. J. (1990]. Examined were the vibration modes' inherent frequencies as well as their shapes. The physical properties of the bone were estimated using the Bayesian approach. Seven natural frequencies were taken out, ranging from 0 to 3000 Hz. The bone was excited with a hammer, and an accelerometer was used to record its response. Dual channel FFT was used to evaluate the transfer function, and the resulting value can be used to determine the natural frequency. The theoretical analysis was based on the Timoshenko beam theory. The analysis ignored the effects of twist in principal planes, bone morrow mass, cancellous bone stiffness, and shear deformation. The geometrical properties of the tibia were calculated using 38 sections in order to characterize the variation in these properties along the bone. It was assumed that the finite elements of the beam type were not uniform and were twisted. The beam is composed of two linearly elastic materials and one perfectly elastic material. The tibia vibrates uniformly despite its intricate shape, which may give the impression of complexity [Thomsen, J. J. (1990].

When Béatrice Couteau (1988) examined the frequency of the human femur bone, she specifically took into account the geometrical and material properties. A finite element model was constructed using the DICOM photos from the CT-Scan device, and it was subsequently modified to take into consideration changes in the density distribution and geometrical properties. The finite element results were validated by vibration analysis tests using an accelerometer, dynamic signal analyzer, and impulse hammer. The maximum and minimum principle planes were taken into consideration when analyzing the transverse vibration's RF. There were 4% variations in the outcomes. This study demonstrated that, even when the heterogeneity of bone tissues was taken into consideration, the accuracy of the results did not significantly improve. This led to the conclusion that the natural frequency is not significantly affected by the femur head or the trabecular bone twist. The primary factors that significantly influence the RF are the cross-section properties, trabecular bone mass, bone morrow, and shear deformation. According to this study, finite element models that account for shape irregularity and density distribution can be constructed using DICOM images. The investigation by Taylor et al. (2002) was centered on estimating the mechanical properties of prostheses that are implanted into the femur bone. Monitoring the stability and distribution of stress-strain was crucial in order to assess the state of these implanted prostheses. The CT-Scan device was used to obtain tomography images for the purpose of performing finite element analysis. The approximation of the distribution of mass and shape modeling along the bone and implanted prostheses were eliminated using these images. The cadaveric femur bone was captured via CT-Scan images. The RF for each vibration mode was found using modal analysis. A modal test was used to experimentally evaluate the natural frequencies. The mechanical properties of the bone with implanted prostheses were computed by comparing the finite element results with the obtained experimental results (natural frequency). The resulting natural frequency has a maximum error of less than 7.8% after matching. Using ultrasound technique, the mechanical properties were evaluated, and the error difference between the two methods (axial Young modulus of elasticity and density) was less than 1%.

The relationship between bone density distribution, bone shape, and bone RF was elucidated by Campoli et al. (2014). To explain the effects of changes in bone density and geometry, finite element models were built. Free-free boundary conditions were assumed in the modal analysis. To perform the statistical analysis, the natural frequency of 27 human femur bones was assessed. A model accounting for the variations in geometry and density among the 27 samples was developed through the application of statistical analysis. By making adjustments to the mean shape model while maintaining a constant mean density distribution, the natural frequency for various vibration modes was found. After that, the shape model was modified while keeping the mean density distribution constant. During the analysis, assumptions were made regarding isotropic materials, linear elastic properties, mean density, and mean Young modulus of elasticity. There were found to be twenty resonant frequencies between 900 and 10,000 Hz [A. Gupta and K. M. Tse., 2014]. Finite element analysis was used by Chiu et al. (2017) to determine whether an internally fixated femur was healed through the use of modal analysis. Intramedullary nail (IM) fixation and plate-style fixation were the two fixation techniques that were examined. Both were implanted on a prosthetic femur made of Saw bones by a surgeon. The finite element model was then created by 3D scanning the prosthesis. These two modal analyses of the fixation predicted the stiffness of the system using each fixation and showed which types of modes were most responsive to healing. Furthermore, a variety of cut orientations were modeled in order to evaluate their effect on the femur's natural frequency. The findings showed that once the wound healed to 10% or higher rigidity, the cut's angle stopped mattering. The angle of the cut was the only factor that naturally determined the torsion mode's frequency at 1% stiffness. Any experimental study must replicate the fracture orientation seen in the field because the natural frequency of the IM nail configuration varied by 9.5%. On the other hand, in 4 out of 5 cut orientations with plate fixation, there was no variation in the natural frequency. This suggests that any cut orientation can be used in vibrational studies in the future.

Vibration Analysis using Experimental Procedure

A study by J. N. Campbell et al., 1971 used mechanical vibration analysis to assess how well a human femur nick heals. The vibration frequency levels used for the experiments were low. Rigid body modes of vibration were found to be caused by such frequencies. The femur vibrates in a variety of complex ways at higher frequencies, and many resonant frequencies have been found. The femur was placed under four different conditions in order to evaluate the effects of a fracture: small part removal from the femur head, femur head dispatch, and femur head reattachment. There were noticeable differences in the resonant frequency (RF) between the four experimental setups. The results show that the RF can be used to track the degree of union of the fractured bone.

Jurist and Kianian, 1973, measured the ulna bone RF. Three models were used to illustrate the effects of the boundary condition, accelerometer weight effect, and skin. The difference between the RMS and RF errors was limited to 20%. This mistake arises from disregarding the twist in the principle axis and assuming the bone is uniformly cylindrical. William P. Doherty,et al, 1994 investigated the usefulness of the RF as a bone integrity assessment factor. The human tibia was subjected to a steady-state variable signal in order to assess the RF of the bone and determine how osteoporosis affects it. This evaluation estimated the clinical utility of the RF as a diagnostic factor for osteoporosis disease. The applied force and the bone's displacement due to vibration helped determine the dynamic mass, stiffness, and RF. The results showed that the RF is less sensitive to alterations in the bone than mass and stiffness are. This achievement can be explained by the fact that those factors influence the RF in addition to the geometrical properties. The loss of mass and stiffness brought on by bone disease will affect the RF.

An electromechanical shaker was used by Gerald A. Thompson et al., 1974 to stimulate the ulna bone in the middle of the bone span. Among the calculated results were the resonance frequency and the modulus of elasticity. The study aimed to develop a noninvasive test to predict the healing of a bone fracture by comparing the results of the fractured bone with the corresponding bone on the other side of the body (if the fractured bone RF reached 95% of healthy bone then the fracture assumed to have healed). The problem that arose during the in vivo experiment was the possibility that the results could be impacted by the soft tissue due to excessive damping. This damping would require higher energy load waves to ensure the excitation of the system, but this increase in energy would cause pain and discomfort for the patient. Bone integrity will be severely compromised by osteoporosis, extended space travel, and prolonged bed rest brought on by a disability. The development of an in-vivo test protocol that measures the bone's strength and stiffness using the RF technique was critically needed. Some information (BMD) regarding the integrity of the bone is provided by radioactive test equipment.

A theoretical and experimental investigation on the elastic characteristics and alterations in bone mineral density linked to aging in the human femur was first presented by Viano, 1976. To evaluate the long bone's elastic properties—which could not be determined with clinical test equipment—such a study was required. The elastic properties were evaluated using the RF and mode shape of deformation calculations for each vibration mode. To make the case study simpler, the proximal and distal ends of the femur bone were removed. This was because the shaft was primarily made of compact bone with relatively little geometry variation (it could be thought of as a hollow, uniform cylinder). It was believed that the bone's composition was linearly elastic and isotropic. Timoshenko beam theory, which also considered rotary inertia and shear deformation, was used to create the transfer matrix. We computed the geometrical properties and density using digital tomography. In the experimental section, an optical technique was used to calculate the displacements caused by the bone excitation. These displacements were used to extract the RF and plot the mode shapes. The Young modulus of elasticity and the shear modulus of elasticity were determined iteratively in order to reconcile the theoretical and mathematical results. In the range of 1–20 kHz, sixteen resonant frequencies were found. The results encompassed the age range of 24 to 85. Density fell by 8% and the Young modulus of elasticity deteriorated by 40, or nearly 10%. The shear modulus of elasticity decreased more quickly than the density and Young's modulus of elasticity. The authors suggested conducting additional research on the topic because the shear modulus of elasticity was more susceptible to aging conditions.

Researchers Subrata Saha Roderic and S. Lakes, 1977 looked at how soft tissues affected the accuracy of the impulse test. Due to its high damping characteristic, the in vivo test results were significantly erroneous due to the soft tissue. Different parameters related to the accelerometer waveform were changed based on the simulation method, soft tissue thickness, and pre-applied force between the accelerometer and the bone. The study concluded that using mass to attach the accelerometer led to lower test accuracy and that using a spring to attach the accelerometer will reduce the error caused by soft tissue. The human volunteer experiences pain despite the experiments. The natural frequency also changed in response to a modification in the boundary condition. In the maximum and minimum inertia plane, T. B. Khalil, et al 1981 examined the he RF of the human femur for three distinct modes of shape: torsional, longitudinal, and transverse. This investigation looked at the centroid trajectory, the calculated area cross-section, the inertia place twist, the second moment of inertia in two directions, and the geometric variation in the femur. This was the first study to take into account the distinct identities of compact and trabecular bone. It was believed that the 59 beams that comprised the bone were composed of two distinct materials: trabecular and compact. These beams were thought to be uniformly cylindrical. The Timoshenko beam theory was used to calculate the longitudinal extension theory, the simple torsion of the non-circular cylinder, and the natural frequency for the transverse vibration. The transfer matrix method was the analytical technique used to calculate the RF. The material properties of the trabecular and compact bones were assumed to be constant. The density and young modulus of elasticity were taken from published works. The natural frequency was also extracted using the impulse test method. The bone was placed on soft foam to mimic the free-free boundary conditions. The experimental results showed that there were twenty resonant frequencies between 20 Hz and 8 kHz. The dynamic signal analyzer was unable to identify each RF's mode of vibration. Consequently, by comparing each RF that was derived from the experimental data with the theoretical analysis, the modes of vibration were found. The axial and bending vibrations separate at the femur centerline curvature of the bone, which was missed in this study despite its importance.

A theoretical and experimental study on the mechanical behavior of the human femur under load was carried out by RR. Huiskes, 1982. For the experimental work, strain gauges attached to the femur shaft were utilized. Theoretical analysis was done using popular stress analysis theories. The theoretical analysis's accuracy in relation to the experimental findings rose when the bone materials were thought of as linearly elastic, homogenous, and transversely isotropic. The femur shaft's geometry was thought to be axisymmetric. In order to evaluate the bone's structural dynamics properties (natural frequency, mode shape, and damping ratio), G. Vander Perre et al., 1983 conducted both in vitro and in vivo experiments. To evaluate the effects of the skin, joints, and muscles, three distinct human Tibia types were used. This study showed how important it was to understand and be conscious of specific factors that affect the test's in vivo accuracy. An analysis of vibration was conducted under the assumption of free-free boundary conditions. A single principal vibration plane (270 Hz & 340 Hz) was linked to each of the two transverse vibration modes that were found. When the in vitro and in vivo tests were compared, it became clear that the additional mass of the muscles was mostly to blame for the differences in results, with skin and joints having very little effect. The notable natural frequency difference between fresh and dry bones could be attributed to the lack of bone morrow.

J. C. Misra and S. C. Samanta, 1984 used torsional wave propagation to study the effects of anisotropic behavior and inhomogeneity of the long bone materials. The bone exhibited piezoelectric behavior as a result of the torsional wave propagation creating magnetic and electrical fields. The phase velocity, the attenuation constant within bone media, and the constant for the bone tissues were assessed using analytical and experimental solutions. The effects of soft tissue and non-uniform bone material were considered in the ultrasonic method. P. Cornelissen et al., 1986 looked at the impact of soft tissues and joints on the structural dynamic properties of the human tibia in order to study tibia vibration in vivo. The 44 BRA and IFR techniques were created to assess how well bone fractures heal. In this work, the modal analysis of the tibia was evaluated using these two configurations. The FRA method used a fixed (kneel) free boundary condition and excited the bone with hammer strikes on the free end. The natural frequency of the tibia was determined using a computer and the Fourier transfer function. The leg was mounted in the same fixed-free configuration as in the BRA method. The reason for the constant vibration in the tibia was an electromagnetic shaker. The results of the vibration in both methods were nearly equally impacted by the soft tissue. The skin had very little effect, but the muscles' increased mass had an impact on the damping ratio and natural frequency. There were almost free boundary conditions and very little joint influence.

The impact of boundary conditions on the estimated natural frequency of human tibia was investigated by Cornelissen, M. et al., 1987. The FRI and BRA procedures were used to process the data after the bone was excited. The specimens were fresh tibias and amputations of the leg. Boundary conditions affected the mode shape and the natural frequency significantly. The natural frequencies and mode shapes that were extracted were related to transverse vibration.

The transverse vibration of the tibia was studied mathematically in 1988 by J. C. Misra and S. C. Samanta. The aim of this study was to find a mathematical model that could explain the natural frequency and damping coefficients. The tibia resembled an elongated cone in general shape. Through experimentation, anisotropic behavior, nonhomogeneity, damping, and the mechanical properties of the bone in the principal planes were evaluated. For the impulse test, an accelerometer and an impulse hammer were utilized. This test was designed to find out if the mathematical model could be used. The viscoelastic model used in this study improved the mathematical results of the solution. There were some differences in the results for the fixed-fixed and free-free boundary conditions. This investigation's results were 45% superior to High's. This disparity could have been caused by the boundary conditions and physical property choices made by J. C. Misra and S. C. Samanta, 1988..

In 1990, G. Nikiforidis presented research aimed at developing a diagnostic method for monitoring fracture healing. With the aid of lateral and axial vibration, a mathematical model was introduced in order to investigate the mechanical properties at the fracture location. The fracture introduces shifting in the phase angle diagram and vibration level due to the coupling between the lateral and axial vibration. This shifting can be used to find the non-uniformities in the bone stiffness. The mathematical model was validated through experiment analysis. Tibia from a freshly cut cadaver served as the specimen for the modal test. The results obtained and the methods' verification encourages more research to make it clinically useful.

The response of the human femur bone to vibration in the frequency range of 0-500 Hz was investigated by Thomas, A. M. C. et al. in 1991. Two configurations of boundary conditions were used in the experimental analysis. The hip replacement procedure ends were simulated using fixed-fixed boundary conditions. In the first boundary condition, the femur RF was determined by the boundary condition and the excitation load. Instead of reaching a resonant condition in the second boundary condition, the bone behaved like a lumped mass. The applied load defined the femur's displacement, which increased with increasing load and was independent of RF [Thomas, A. M. et al, 1991]. Natural frequency analysis was used by Lowet et al., 1993 to evaluate the torsional stiffness of different animal bones. Experiments were conducted to validate a basic beam model that is used to calculate the torsional stiffness of long bones. A simple mathematical model for the relationship between torsional stiffness and natural frequency was developed using the torsional impact test. The response of the human femur bone to vibration in the frequency range of 0-500 Hz was investigated by Thomas, A. M. C. et al. in 1991. Two configurations of boundary conditions were used in the experimental analysis. The hip replacement procedure ends were simulated using fixed-fixed boundary conditions. In the first boundary condition, the femur RF was determined by the boundary condition and the excitation load. Instead of reaching a resonant condition in the second boundary condition, the bone behaved like a lumped mass. The applied load defined the femur's displacement, which increased with increasing load and was independent of RF [Tower, S. S., et al, 1993].

A study to evaluate the mechanical characteristics in a cohort of patients infected with osteoporosis was presented by Van Der Perre et al. [32]. The effect of aging on the mechanical properties of bone was added to the study. Vibration analysis and ultrasonography were used to evaluate the mechanical properties of human and animal bones. Basic beam theory was used to explain the relationship between the natural frequency and geometrical and physical properties. An analysis of the mathematical model's validity was done, drawing on past findings regarding animal bones. A sample of patients and a group of patients of different ages were gathered in order to conduct the in vivo test. The bending rigidity, as assessed by vibration analysis, decreased and had lower values in the osteoporosis patients than in the age control group, according to the results of the vibrational and ultrasonography in vivo tests. The ultrasonic velocity of the age control group was higher than that of the osteoporosis patients. Alterations in the physical properties of bone tissues could be linked to this fluctuation in ultrasonic speed. On the other hand, osteoporosis will cause a shift in bending rigidity without significantly changing the tissue's material composition. By combining these methods, the entire picture of how osteoporosis and aging impacted the mechanical properties of the tibia bone could be seen. Those noninvasive tests can be used in conjunction with the imaging technique to develop a clinically useful device for evaluating bone integrity. The impulse response method was employed by Nakatsuchi Y. (1996)

to evaluate the fractured bone's bending rigidity during the healing process. Fifty-five percent of the samples showed a temporal decrease in the natural frequency during the first stages of the treatment. A similar decline was noted in 80% of external fixation treatment procedures. There was an additional decrease after the external fixation was removed. In contrast to the contralateral side, the natural frequency was higher during the final phase of treatment. The accuracy of tracking fracture healing was very good in 58% of the examination cases, good in 31%, and poor in 11% of cases. According to the research findings, the impulse response method can be very helpful in the early stages of treatment for determining the bending rigidity of fractured bone [Nakatsuchi Y and Tsuchikane A, Nomura A, 1996].

In order to evaluate bone strength, Pinar Arpinar et al. (2005) reported a study that used vibration analysis and dual energy x-ray absorptiometry. The dual energy x-ray absorptiometry (DEXA) measurement of bone mineral density has certain limitations when evaluating the mechanical properties of the bones. Therefore, a noninvasive method was required to accurately evaluate how changes in bone strength (osteoporosis) caused by metabolic diseases. The bone mineral density ascertained by DEXA was compared with the outcomes of the modal analysis. Seven sets of female tibia made up the case study samples. Seven mothers and their daughters took part in the in vivo analysis of the left and right tibia bones. The bone mineral density of each sample was determined using vibration analysis and DEXA. A relationship was discovered between the bone mineral densities computed using the two techniques. Compared to the daughter's sample, the mother's had a higher RF and ultrasound velocity. Compared to the mother, the daughter's bone sample was stronger. The study's findings suggested that a technique for forecasting bone strength might be created using vibration analysis and the correlation between the bone mineral densities determined by DEXA.

A study was carried out by G.E. Christopoulou et al. (2006) to assess the efficacy of modal damping as a factor in the diagnosis of osteoporosis. Fifteen adult female rates were used in the experiment. Osteoporosis was allowed to progress at these rates for sixty days before alendronate was given as a treatment. The rates were ascertained by assessing the bone pathological health conditions (bone mineral density) prior to, during, and subsequent to treatment using a CT scan. The modal damping factor was found through a model test. The rate's tibia bones were stimulated in vivo, and the accelerometer helped collect the response signal after that. The amplified signal from the latter source was fed into a dynamic signal analyzer. The fast furrier transform was used to assess the resonant frequencies. The quality factor (Q) can be used to calculate the modal damping factor after the peaks have been identified. The change in the model damping factor was either greater than or equal to the change in trabecular bone. The modal damping factor increased as the density of the trabecular bone decreased. The results of the study indicate that osteoporosis can be diagnosed and the healing process evaluated using the model damping factor. Gregory N. (2008) presented a study that used frequency analysis to evaluate the structural integrity of the long bones in order to determine the integrity of the spine. Using cadaveric pigs, modal analysis yielded five spin samples. To excite the spin, a brief vibration signal (0-2000 Hz) was applied at the L3 spinous end. The response was collected using tri-axial accelerometers fastened to the L1–L5 spinous branches. A dynamic signal analyzer was used to draw the frequency response function

(FRF) for each accelerometer. The structure was changed to collect more data by joining adjacent spinous (FRF). Coherence was used as a proxy for the quality of the modal test. According to the investigation, the FRF is very sensitive to changes in the spin structure and can be used as a diagnostic tool.

In an in vitro and in vivo study, Bekir Bediz (2010) employed vibration analysis to evaluate alterations in the structural dynamic properties of the human tibia. Dual energy x-ray absorptiometry was used to measure the bone mineral density of the bone samples and the volunteer tibia (in vivo and in vitro). The influence of skin and soft tissue on the mechanical vibration was considered. The modal test was conducted using an accelerometer, an impulse hammer, and a dynamic signal analyzer. The results showed that the RF of the tibia decreased in tandem with a decrease in bone mineral density. Muscles reduced the chance of bone excitation in the in vivo test without producing pain. They had a bigger effect than the skin, fibula, and the challenge of figuring out the vibrational modes. On the other hand, there was a noticeable difference in the modal damping (loos factor) when the collagen and bone mineral density were reduced. The results of this study provided support for the application of modal damping as a diagnostic method for metabolic bone disorders and the healing of bone fractures. A noninvasive diagnostic technique to differentiate between pathological and normal health conditions can be created by applying RF and modal damping. Vibration analysis was used by van Engelen (2012) to predict the mechanical properties of human lumber motion segments. The ability of small forces and deflections to induce vibration and evaluate the integrity of the structure encouraged the development of such a technique to determine the pathological health condition of the human spin. The purpose of this study was to investigate the connection between vibration analysis and quasi-static mechanical testing. Quasi-static mechanical testing (in vitro L1-L5 spinous) was performed on six human spins. The spin segments' angular deflections were calculated using an optical device. To create consistent vibration, a shaker was applied to the upper vertebra. The response was recorded using tri-axial accelerometers in the mediolateral and anteroposterior planes. Plotting the frequency response function revealed the RF and mode shape. The natural frequency for the transverse in two main plans—axial vibration and torsional vibration—was extracted with extremely high accuracy. Based on the outcomes of the two tests, a strong correlation ($r = 0.7$) was discovered between the two methods. Vibration analysis was found to be a useful tool for evaluating the mechanical properties of human lumber motion segments.

The relationship between bone density distribution, bone shape, and bone RF was elucidated by Campoli et al. (2014). To explain the effects of changes in bone density and geometry, finite element models were built. Free-free boundary conditions were assumed in the modal analysis. To perform the statistical analysis, the natural frequency of 27 human femur bones was assessed. A model accounting for the variations in geometry and density among the 27 samples was developed through the application of statistical analysis. By making adjustments to the mean shape model while maintaining a constant mean density distribution, the natural frequency for various vibration modes was found. After that, the shape model was modified while keeping the mean density distribution constant. During the analysis, assumptions were made regarding isotropic materials, linear elastic properties, mean density, and mean Young modulus of elasticity. There were found to be twenty resonant frequencies between 900 and 10,000 Hz. Lorenza Mattei et al. (2016) investigated the calculation of the shift in the structure's fixation natural frequency caused by the healing process in order to evaluate fracture healing. Because the fixation structure was in direct contact with the bone, the vibration analysis was not as significantly dampened by soft tissues and skins. Additionally, the fixation structure's pins functioned as points of excitation and response collection. The accelerometers were easier to install because these were level locations. It was established and confirmed the natural frequency of the bone and fixation structure (as a single system) and detailed how natural frequency is impacted by the healing process. The Tibia phantom was used in eighty tests to evaluate the experimental protocol. Various setups were used to validate the tests by analyzing the natural frequency for the tibia phantom alone, tibia with pins, and the entire fixation structure with tibia phantom. The structure was excited using a micro-hammer, and the response was measured using accelerometers. The signals that were gathered and excited using pins were used to plot the frequency response function. In vivo tests were conducted on volunteers, and the natural frequency increased as the fractured bone healed.

With an emphasis on fractures treated with external fixators, Mattei et al. (2017) reexamined the frequency response analysis quantitative method for healing assessments. The solution to the problems of soft tissue transmission and damping was found in the application of fixator pins to accomplish both bone excitation and response. The main objective was to develop and validate a test protocol to characterize the treated bone. More than 80 tests were performed on tibia phantoms without pins, phantoms with pins, and phantoms with a complete fixator. Different input-output combinations and excitation methods were compared. The results demonstrated the effectiveness of a technique based on impact tests using a micro-hammer. It was demonstrated that by increasing the number of resonant frequencies, fixators and pins could change the phantom's frequency response.

Di Puccio et al. (2017) proposed a robust protocol for bone healing detection through vibrational methods. A tibia phantom was used for the first test campaign. A similar procedure was then applied to the same phantom during a period of simulated healing. Two case studies were used for the in-vivo tests, with one having a tibia that was in good health and the other having a significant difference. The results showed that variations in the resonant frequencies could be actually observed every two weeks, which allowed for a more frequent control with respect to the real X-ray solution.

Di Puccio et al. (2017) focused on fractures treated with an external fixator when examining the application of experimental modal analysis to fracture healing assessment. The goal was to find out if, in the presence of the fixator, which might obfuscate the bone response, changes in the resonant frequencies could be interpreted as changes in the bone-callus stiffness. Using glue and resin, three different callus surrogates were made to mimic the in vitro healing process on a tibia phantom. The resonant frequencies of the phantom with the entire fixator and the phantom with screwed pins were estimated. The results supported the use of experimental modal analysis in fracture healing monitoring by validating the observation that the frequencies increased with increased simulated bone-callus stiffness. This approach offered a number of advantages over the real standards, including the ability to monitor the healing process more frequently due to its quantitative and non-invasive nature.

A vibration technique was proposed by Mattei et al. (2018) to monitor the healing process of an elongated femur treated with an external fixator. Impact testing (IT) stimulated the bone and recorded its response using pins that screwed into it. As a result, transmission through the soft tissues was halted and signal quality was raised. Impact tests were carried out every three to four weeks for a duration of five months. Unfortunately, after seven weeks, an infection required the removal of multiple pins, requiring changes to the system. Two different configurations were considered both before and after pin removal. In the final two sessions, a different arrangement was used in which the fixator body was removed and four pins were left in the femur. The evolution of the system's resonant frequencies and frequency response function was studied during the monitoring period. The information provided by X-ray images was compared with the IT findings. The system's resonant frequencies were found to rise by approximately 2-3% every week while the callus transitioned from the soft phase to the woven bone. The largest increase (approximately 22%) was seen in the first RF. After the woven bone formed, the vibratory response remained almost unchanged, suggesting that the healing assessment could be affected by the relative variation in the resonant frequencies. The results demonstrated the value of the IT approach as a tool for evaluating fracture healing.

A human tibia replica with identical mechanical and morphological characteristics was dynamically characterized by Verdenelli et al. (2018) in both the presence and absence of an external fixation system. In this study, FRFs and modal parameters were compared between the tibia alone and in the presence of an external fixation system. Furthermore, a comparison was performed between a numerical model of these structures and the experimental data. A laser Doppler vibrbrometer (632 nm wavelength) was used for the non-contact approach, and tests were carried out with a mono-axial accelerometer for the contact measurement approaches. To simulate free-free conditions, the tibia was positioned on a foam support. A shaker and a micro-hammer supplied the input in the same excitation direction. The results suggested that both shaker-based excitation and the micro-hammer excitation method could cause issues with the response measured with lasers. Modal analysis results tended to mitigate the problems caused by the tendency of laser-based data to sense a vertical in-plane in correspondence of the third horizontal mode, since multiple components were sensed simultaneously during the scan.

In their 2019 study, Mattei et al, showed that measuring bone healing in in vivo fractures treated with external fixation can be achieved effectively with mechanical vibration. A patient with a tibial fracture in the case study received treatment with a monoaxial fixator. Over a period of three months, five impact tests were used to monitor the healing process. The pins that were screwed into the bone were used to both stimulate and measure the vibrations. Fracture healing could be quantitatively assessed by estimating the leg's resonant frequencies. The first frequency increased by about 4% per week during the observation period. After the 13-week period of hard callus formation, other frequencies increased by 1% to 6% each week. X-ray observations proved the method's viability and potential for the healing process. After the fixator was removed, the vibratory response was evaluated and five modes in the 0–1000 Hz bandwidth were found. The results suggest that a vibratory response from a fractured bone treated with external fixation may be a valuable indicator for quantitative healing monitoring. The authors came to the conclusion that using the mechanical vibration method more frequently would be preferable and that it might help patients receive less X-ray exposure.

In their 2019 study, Chiu et al, investigated the application of vibrational analysis to monitor the healing status of a femur fixed with a plate-screw in addition to the current clinical radiographic assessment. In order to mimic the damping effect of soft tissues above, an osteotomized composite femur specimen covered in modeling clay was used in this experiment. Epoxy adhesives were used to cover the fractured area in order to simulate the healing process. With the equipment mentioned above, the cross-spectrum and coherence were acquired, and they were then examined over time in the frequency domain. The results suggested that the cross-spectrum and the proposed healing index needed to be analyzed in order to quantitatively assess the stages of healing. The results also showed that the mass loading effect brought on by modeling clay had no effect on the recommended healing assessment method. The findings suggested a potential non-invasive technique for evaluating the rate of healing of a femur fracture using vibrational responses.

Mattei et al. (2021) used a quantitative, non-invasive vibrational method to assess the healing of a complex fracture treated with external fixation. Following surgery, callus stiffening was monitored until the fixator was removed. Their approach, which included more frequent testing (bi-weekly), a longer healing monitoring period (roughly nine months), and an analysis of a single test configuration, overcame the earlier shortcomings of the other health testing techniques. The healing process was observed by looking at the percentage increases of the squared resonant frequencies (SFIs), which were related to the changes in stiffness and the frequency response functions. The first resonance frequency was found to be the most sensitive parameter for measuring healing, as supported by X-ray images. When the woven callus was forming, it increased by about 20% per month, and when the healing process was finished, it increased by up to 50%. Their research confirmed the vibrational method's potential to replace radiography in the assessment of fracture healing.

Vien et al. (2022) reported a clinical study that examined modal frequencies associations with human leg musculoskeletal components using a prototype device based on a vibration analysis method. The first out-of-plane and coupled modes in the frequency range of 60 to 110 Hz were found to be correlated with femur length, suggesting that these modes were suitable quantitative markers for evaluating bone health. In addition, stepwise regression models derived through mathematics were employed with the measured leg components as variables in order to determine the modal frequencies. Femur length was one of the independent variables in the first mode, and it explains about 43% of the variation in modal frequencies. The findings provided direction for further investigation and advancement concerning the utilization of vibration-based methods for accurate monitoring of bone and fracture healing.

OBSTACLES AND PROSPECTS FOR APPLYING FREQUANCY RESPONSE AND VIBRATION ANALYSIS FOR BONE HEALING EVALUATION

Although bone healing assessment using vibration analysis and frequency response techniques shows promise, there are a number of obstacles to overcome and more research to be done before these methods can be made more effective. The following are some of the main issues and potential paths in this field:

Standardization and Validation: To prove the accuracy and dependability of vibration analysis and frequency response techniques for bone healing assessment, validation studies and standardized protocols are required. Standardization would guarantee uniformity in the collection, evaluation, and interpretation of data among various research projects and medical environments. Large sample sizes and a variety of patient populations would be useful in validation studies to determine the clinical utility and efficacy of these methods.

Soft Tissue Interference: The accuracy of assessments can be impacted by soft tissues like muscle and skin, which can affect how vibrations are transmitted. The vibration signals may contain noise and artifacts due to the coupling between the soft tissues and bone. Subsequent investigations ought to concentrate on devising strategies to mitigate the impacts of soft tissue disruption, like employing specific attachment methods or signal processing algorithms to separate the response of the bone from the adjacent tissues.

Individual Variability: The characteristics of bones and the processes involved in their healing vary greatly amongst individuals. Vibration analysis response can be affected by age, sex, bone density, and comorbidities, among other factors. Subsequent research endeavors ought to examine the influence of these distinct variables on vibration-based evaluations and devise customized methods for assessing bone healing. Machine learning.

Early Detection and Monitoring: Timely intervention and better results depend on the early detection and monitoring of bone healing. The sensitivity of traditional methods to identify early healing changes is frequently compromised. Future studies should concentrate on creating sensitive vibration analysis methods that can recognize early indications of bone healing and monitor the healing process over time. For increased sensitivity, this might entail investigating unique vibration parameters or integrating vibration analysis with other imaging modalities.

Tracking the healing trajectory and evaluating the efficacy of interventions require longitudinal monitoring of bone healing. Techniques for frequency response and vibration analysis may be able to offer real-time monitoring. But issues like device portability, patient compliance, and data analysis techniques must be taken into the considerations.

Clinical Applications and Integration: Although experimental studies have constituted a large portion of this field's research, future endeavors should concentrate on integrating vibration analysis and frequency response techniques into clinical settings. This would entail carrying out extensive clinical trials to assess these methods' efficacy in different bone-healing situations, like fractures, non-unions, and osteoporosis. Furthermore, a thorough assessment of bone healing might be possible by combining vibration analysis with the clinical assessment instruments and imaging modalities currently in use.

Applications in Therapy: Vibration methods have the potential to improve bone healing in a therapeutic setting beyond evaluation. It has been demonstrated that low-intensity vibration promotes bone formation and quickens the healing process. Future studies should examine the therapeutic uses of vibration methods, such as creating focused vibration treatments.

CONCLUSIONS

As a useful tool for evaluating bone healing, frequency analysis techniques provide objective, quantitative evaluations of the structural integrity and properties of the bone. Frequency analysis is a desirable alternative for tracking the development of bone healing processes because of its non-invasiveness and capacity to monitor changes in bone healing over time. Vibration techniques reveal information about bone stiffness, damping characteristics, and changes in bone mineral density through the examination of resonant frequencies and frequency response patterns. Making decisions about treatment can be aided by these parameters, which act as markers of the state of bone healing. While there are certain obstacles and restrictions related to the application of frequency analysis in the evaluation of bone healing, such as the impact of soft tissue interference and the requirement for standardization, these issues are being addressed by continued research and developments. To reduce the influence of confounding variables, efforts are being made to create standardized protocols, confirm the efficacy of these methods, and investigate cutting-edge strategies. Frequency analysis techniques have potential uses in therapy to aid in bone healing, so their utility goes beyond evaluation. It has been demonstrated that low-intensity vibration can promote bone formation and quicken the healing process. Targeted vibration interventions could be developed as a result of more research in this field to maximize bone healing results. To sum up, frequency analysis methods provide a quantitative and non-invasive way to evaluate bone healing. These methods have great potential to advance therapeutic interventions in the field of bone healing, facilitate longitudinal monitoring, and improve clinical decision-making, even though more study and validation are required. Frequency analysis techniques can lead to better patient outcomes and a better understanding of the bone healing process with further advancements and integration into clinical practice.

Figures

Fig. 1.2 Dual-Energy X-ray Absorptiometry [5]

Fig. 1.3 QCT devise

Fig. 1.4 Quantitative Ultrasound (QUS) [6]

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