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LATERAL-TORSIONAL BUCKLING OF THIN-WALLED SHEET USED AS COLD-FORMED BEAM

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ABSTRACT:

The use of cold-formed steel members as structural columns and beams in residential, industrial and commercial building was increased significantly in recent time. This study is focused on the use of cold-formed steel sections as flexural member subjected to lateral torsional buckling. For this purpose a software **ANSYS** program was developed and validated using available numerical and experimental results. This paper presented the results of finite element analysis for cold-formed channel cross-section and the comparisons with **AISC** and **Australian** design rules. Failure in cold-formed steel beams is generally initiated by one of three instabilities: local, distortional, or lateral-torsional buckling. The results indicated that the value of maximum ultimate load occurs due to point load act on the shear center, the percent of increasing about (35.84%). Due to differences in the degree of cold working in the flat parts and the corner regions, the mechanical properties vary over the cross-section. In cold-formed channel beam, the lateral torsional buckling moment occurs between elastic and inelastic critical moment . The convergence about (97.83%) with the experimental data and (89.78%) with the numerical results.

KEY WORD: lateral torsional buckling, cold-formed channel cross-section.

الخلاصة:

استعمال Cold-formed كأعمدة وعتبات هيكلية في البناية الصناعية ، التجارية والسكنية زِيدث بشكل ملحو في الوقت الأخير. ركزت هذه الدراسة على استعمال Cold-formed كأعضاء flexural معرضة إلى انبعاج اللي الجانبي. لهذا الغرض طور برنامج ال ANSYS لتحليل مقاطع cold-formed channel باستخدام العناصر المحددة ومقارنة النتائج ع AISC وكذلك Australian design rules بالإضافة إلى النتائج العددية التي تم الحصول عليها من المعادلات المقدمة في هذا البحث بالإضافة إلى النتائج العملية المتوفرة والمقدمة من قبل عدد من الباحثين . الفشل في مثل هذه المقاطع تبدأ عموما ب(distortional local ، أو lateral-torsional buckling). أشارت النتائج بان قيمة الحمل النهائي الأقصى يحدث عند تسليط الأحمال في الshear center بزيادة (35.84%) وبسبب الاختلاف في درجة العمل لتصنيع ال (Cold-formed) في الأجزاء المستوية ومناطق الزاوية تفاوتت الصفات الميكانيكية على المقطع العرضي. أما نسبة التقارب بالنتائج العملية بما يعادل (97.83%) و (89.78%) بالنتائج العددية.

INTRODUCTION

Thin sheet steel products are extensively used in building industry, and range from purlins to roof sheeting and floor decking. Generally these are available for use as basic building elements for assembly at site or as prefabricated frames or panels. These thin steel sections are cold-formed, i.e. their manufacturing process involves forming steel sections in a cold state (i.e. without application of heat) from steel sheets of uniform thickness. These are given the generic title Cold Formed Steel Sections. Sometimes they are also called Light Gauge Steel Sections or Cold Rolled Steel Sections.

A theoretical and experimental study of the buckling behavior of channel beams with unbraced longitudinal edge stiffeners, bent in such a way that the stiffeners are in compression, is presented by [Seah and Khong, 1990]. For a relatively short length of beam loaded in such a manner, the beam will buckle symmetrically. However, for a relatively long beam the lateral-torsional buckling may well be the governing mode of failure.

The AISI Direct Strength Method (DSM) was introduced in 2004 and uses the elastic buckling properties of the entire cross-section to calculate the capacity [Schafer 2002]. The advantages of this method can be seen in the design of members with complexes cross sections, for example intermediate and edge stiffeners, where the calculations based upon the effective width method would be cumbersome and time consuming.

Lateral torsional buckling failure is the most complex failure criterion of materail beam. This type of failure is identified by the simultaneous bending and twisting of the entire cross-section . If a beam is not restrained laterally, it tends to fail by lateral torsional buckling in cases where lateral stiffness and torsional stiffness are low[Chu, et. al.,2004]. Buckling modes assumed to control capacity are local buckling, distortional buckling and global buckling (lateral torsional buckling in the beams)[Schafer and Adány 2006].

The cold-forming process can alter the mechanical properties of cold-formed steel from the parent steel sheet. The cold-forming process increase both the yield strength and the ultimate tensile strength while reducing the ductility [Yu,2000, Chen and Young,2006]. However, according to [Yu,2000, Chen and Young,2006], the percentage increase in tensile strength is much smaller than that of yield strength with a consequent marked reduction in the difference between yield point and tensile strength.

The behavior of cold-formed steel lipped channel beams subject to local and lateral-torsional buckling effects at both ambient and elevated temperatures[Kankanmge, 2010].

TYPES OF BUCKLING

Cold-formed steel beams are commonly used in civil construction as both secondary, e.g., grits and purlins, and primary, e.g., floor joists, structural members. This paper focuses on one of the most common sections employed in these applications, the C section. Cold-formed steel C sections are formed from coils of thin metal (on the order of 1 mm thick) and the resulting cross section is thin-walled. Thin-walled members must carefully consider the role of cross-section instability in their design. Cross-section instabilities in C section beams include: local buckling, distortional buckling, and lateral-torsional buckling. The cross-section deformations associated with each of the three buckling modes are illustrated in Fig.1. Local buckling involves distortion of the cross section with only rotation occurring at interior fold lines of the section. Distortional buckling involves distortion of the cross section with rotation and translation occurring at interior fold lines. Lateral-torsional buckling excludes distortion of the cross section; however, translation and rotation of the entire cross section occur. The local, distortional, and lateral-torsional buckling modes also differ greatly in their longitudinal variation along the beam. The longitudinal deformation associated with each of the three buckling modes is sinusoidal with a half-wavelength as identified by the minima in Fig.1.The local buckling mode occurs with repeated waves at a short length, while lateral-torsional buckling occurs in one half-wave over the unbraced length of the beam. Distortional buckling repeats at a wavelength intermediate to the two other modes. The moments associated with each of the three buckling modes are given in Fig.1 as the ratio of the elastic critical buckling moment (M_{cr}) to the moment at first yield (M_y). The minima in Fig.1 provide the critical values. Determination of the bending strength, for use in design, requires consideration of these cross-section instabilities, as well as the differing post-buckling characteristics in each of the buckling modes, potential interaction amongst the modes, and material yielding.

PREDICTING RESIDUAL STRESSES AND PLASTIC STRAINS IN COLD-FORMED STEEL MEMBERS [Cristopher and Schafer 2009]

Thin cold-formed steel members begin as thick, molten, hot steel slabs. Each slab is typically hot-rolled, cold-reduced and annealed before coiling and shipping the thin steel sheet to roll-forming procedures[US Steel 1985]. The measured surface strains are converted to residual stresses using Hooke's Law and then distributed through the thickness as membrane (constant) and bending (linear variation)components. These residual stress distributions are a convenient way to express the measured residual surface strains, and are convenient as well for use in nonlinear finite element analysis, but they are not necessarily consistent with the underlying mechanics. Plastic bending, followed by elastic spring back, creates a nonlinear through thickness residual distribution, in the direction of bending, as shown in Fig.2. [Shanley 1957].The presenceof nonlinear residualstress distributions incold-formedsteel members has been confirmed inexperiments[Keyand Hancock 1993]press-braking steelsheets[Quach et al. 2006].

A closed-form analytical prediction method for residual stresses and equivalent plastic strains from coiling, uncoiling, and mechanical flattening of sheet steel has also been proposed [Quach et al. 2004].The same plastic bending that creates these residual stresses also initiates the cold-work of forming effect where plastic strains increase the apparent yield stress in the steel sheet and ultimate strength in some cases[Yu 2000].

STRUCTURAL PERFORMANCE:

According to Europe Code[ENV 1993-1-1:1992/A1:1994], welding of cold formed sections should not be carried out in the cold deformed zones or within the adjacent width of (5t) each side, see Table 1 , unless either:

- The cold –formed zones are normalized after cold-forming but before welding,
- The thickness does not exceed the relevant value obtained from Table1,[Products Hand Book, Structural Steel]

Due to stress relief effects, cold formed hollow sections are subject to greater distortion than hot finished section when subject to shot blasting, galvanizing and welding. This can cause local buckling, corner cracking and other deformations, and will obviously have a large impact on the capacity when used as beams and columns.

NUMERICAL ANALSIS FOR LATERAL TORSIONAL BUCKLING:

The lateral torsional buckling behavior is illustrated in Fig.3, relating to buckling resistance and slenderness. There are three different ranges of behavior namely, elastic buckling, inelastic buckling and plastic behavior. The elastic lateral torsional buckling occurs in slender beams with low resistance to lateral bending and twisting. As the slenderness decreases, the resistance of abeam to undergo elastic buckling increases and the beam may yield before its elastic buckling moment is reached. Yielding reduces the effective out-of-plane rigidities, and hence, lateral torsional buckling occurs before reaching the elastic buckling moment. This type of buckling of beams having intermediate slenderness is called the inelastic lateral torsional buckling. If the beam is fully or adequately restrained laterally so that the slenderness is low, it achieves the full plastic moment capacity.

The elastic critical buckling moment for lateral torsional buckling is given next . [Hill,1954, Trahair,1993, Trahair and Bradford,1988,Yu,2000].

$$M_{cr} = \sqrt{\left(\frac{\pi^2 EI_y}{L^2}\right)\left(GJ + \frac{\pi^2 EI_w}{L^2}\right)} \tag{1}$$

Where:

EI_y ,GJ and EI_w are the minor axis flexural rigidity, torsional rigidity and warping rigidity, respectively.

The lateral torsional buckling is given next.[Trahair,1993,Pi and Trahair,1992 a,b].

$$M = \frac{M_{cr}}{\sqrt{\left(1-\frac{EI_y}{EI_x}\right)\left(1-\frac{\left[GJ+\frac{\pi^2 EI_w}{L^2}\right]}{2EI_x}\right)}} \tag{2}$$

The inelastic critical buckling moment for lateral torsional buckling is given by [Trahair and Bradford,1988].

$$M_{cr} = \sqrt{\left(\frac{\pi^2 \left(EI_y\right)_e \left(GJ\right)_e}{L^2}\right)\left(1+\frac{\pi^2\left(EI_w\right)_e}{\left(GJ\right)_e L^2}\right)} \tag{3}$$

a) AISC

The nominal flexural strength, M_n , shall be the lower value obtained according to the limit states of yielding(plastic moment) and lateral torsional buckling[AISC].

1. yielding

$$M_n = M_p = F_y Z_x \tag{4}$$

Where:

F_y = specified minimum yield stress of the type of steel being used,(MPa).

Z_x =plastic section modulus about the x-axis,(mm^3).

2 . lateral- torsional buckling

a) when $L_p \le L_r$, the limit state of lateral- torsional buckling does not apply.

b) when $L_p \le L_b \le L_r$,

$$M_n = C_b \left[M_p - \left(M_p - 0.75 F_y S_x \right) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \le M_p \qquad (5)$$

c) when $L_p > L_r$

$$M_n = F_{cr} S_x \le M_p \qquad (6)$$

Where:

L_b = length between points that are either braced against lateral displacement of compression flange or braced against twist of the cross section,(mm).

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{st}} \right)^2} \sqrt{1 + .078 \frac{JC}{S_x h_o} \left(\frac{L_b}{r_{st}} \right)^2} \qquad (7)$$

where:

E =modulus of elasticity of steel, (MPa).

J =torsional constant,(mm^4).

S_x =elastic section modulus taken about the x-axis,(mm^3).

The limiting lengths L_p and L_r are determined as follows:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \qquad (8)$$

$$L_r = 1.95 r_{st} \frac{E}{0.7 F_y} \sqrt{\frac{JC}{S_x h_o}} \sqrt{1 + \sqrt{6.76 \left(\frac{0.7 F_y}{E} \frac{S_x h_o}{JC} \right)^2}} \qquad (9)$$

Where:

$$r_{st}{}^{2} = \frac{\sqrt{I_y C_w}}{S_x} \tag{10}$$

And

$$C = \frac{h_o}{2}\sqrt{\frac{I_y}{C_w}} \tag{11}$$

Where:

h_o =distance between the flange centroids,(mm).

C_w= warping constant,(mm^6).

b) AS/NZS 4600

The nominal member moment capacity (Mb) of the laterally unbraced segments of singly, doubly, and point-symmetric sections subjected to lateral buckling is given by **AS/NZS 4600**,

$$M_b = Z_c\left(\frac{M_c}{Z_f}\right) \tag{12}$$

Where

Z_c and Z_f are the effective section modulus calculated at a stress level (M_c/Z_f) in the compression fiber, and the full unreduced section modulus for the extreme compression fiber, respectively. The critical moment (Mc) can be calculated as follows,

For $\lambda_b \le 0.6$ $\qquad M_c = M_y$

For $0.6 < \lambda_b < 1.336$ $\qquad M_c = 1.11 M_y\left[1 - \frac{10\lambda_b{}^2}{36}\right]$

For $\lambda_b \ge 1.336$ $\qquad M_c = \frac{M_y}{\lambda_b{}^2}$ (13)

Where $\lambda_b = \sqrt{M_y / M_o}$ is the non-dimensional slenderness ratio. $M_y = Z_f\ F_y$ is the moment causing initial yield at the extreme compression fiber of the full section.

FINITE ELEMENT MODEL AND ANALYSIS

Finite element model

At present, finite element analysis programs and extensively used and have greater importance in the field of research. This trend is increasing as the use of finite element analysis programs is relatively inexpensive and time efficient compared with large number of full scale tests. The finite element analysis program **[ANSYS]** is a very important tool that is widely used in engineering applications. It can be used to solve problems ranging from relatively simple linear analysis to the complex non-linear analysis. A nonlinear finite element modal using **[ANSYS]** has been developed to investigate a cold-formed steel channel cross section with a thickness of (6 mm) and having the dimensions of web depth (160) mm and flange width (70) mm .A three cases of point load was applied to the one end of cantilever beam across the section as shown in **Fig.4.**

Finite element analysis using **[ANSYS]** usually consists of two major stages: Eign-value buckling analysis to predicted the mode shape and the non-linear buckling analysis to calculate the ultimate loads. **Figs. 5,6 and 7** show the deformed shape of lateral- torsional buckling for three cases of loads acting on the cantilever beam as shown in **Fig. 4.** From these figs. results , there are two types of global initial geometric imperfections may exist in mono-symmetric cold-formed steel channel section beams, namely negative and positive imperfections. The negative imperfection is identified by the lateral movement of the cross-section outward with anticlockwise twist as shown in **Fig. 8,a** while the cross-section deformed in ward with a clockwise twist on the case of positive imperfection as shown in **Fig.8,b.** The direction and magnitude of initial geometric imperfection present in beam is very important especially for mono-symmetric sections as its effect on the ultimate failure capacity is considerably high **[Pi et al,1997]**.

APPLICATION

Cantilever Beam

A channel cross-section of cold-formed steel beams made of (6mm) and (10mm) thick G450 steels with (70mm) width of flange and (160mm) depth of web for cantilever beam subjected to three cases of point load acting on the free end **Fig.4.** The results plotted in **Figs.9**, maximum ultimate load occur due to point load act on the shear centre the percent of increasing about (35.84%).

For the same section , the initial material state is sometimes considered through the so-called cold-work of forming effect, where the yield stress of the material is increased in corners will reach the fully plastic stress compared with flatting edges is lower see, **Table 1**. In main while, two grade of steel stress used in the same section to compared with one grade of steel stress used as shown in **Fig.10.** From the results the convergence between two cases about (59.42%) and cold forming is known to increase the yield and ultimate strength of the material due to cold working or strain hardening.

Higher ultimate load capacities were achieved in the higher thickness and t_w is not equal to t_f for the same area of cross section under the same conditions of loading. The results plotted in **Fig. 11** shows the convergence between two cases about (41.07%) .

The effect of long for cantilever beam were also investigated. **Figs. 12 and 13**, show these results for (1.5m) and (2.8m). The maximum ultimate strength of these two cases are different about (62.19%)

Simply supported beam.

In case of cold- formed steel of simply supported beam subjected to two point loads acting on the shear center of channel cross section was used in the cantilever beam as before. **Figs.14 and 15** show the comparison between finite element analysis (present study) and the numerical results related to **AS/NZS 4600 [SA,2005]** formulas and **AISC ,**respectively**.**

From **Fig.16** it can be seen that curve fit well the experimental values. However, it can be seen be from **Fig.16** that for cold-formed channel beam the lateral torsional buckling moment in between elastic and inelastic critical moment compared with **[Trahair 1993]** numerical equations.

The value of lateral torsional buckling moment from **ANSYS** is equal to (2.3 kN.m) and convergence about (97.83%) with the experimental values and (89.78%) with **[Trahair 1993]** numerical equations.

Mechanical Properties of cold-formed steel- channel section.

Generally, the depth of cold-formed steel section (160mm), the width (70mm) and (6mm) thickness. The nominal yield strength of cold-formed steels ranges from (250 to 550 MPa.) while their modulus of elasticity is 210000 MPa.

Due to differences in the degree of cold working in the flat parts and the corner regions, the mechanical properties vary over the cross-section as shown in **Fig. 17**.

CONCLUTIOS

This paper was based on the results of a finite element analysis based parametric study into the lateral torsional buckling behavior of cantilever beam cold-formed and simply supported beam cold-formed steel flexural member subjected to concentrated load.

From the FEM analysis of many problems of cold-formed channel cross section of beam, many conclusions can be drawn:

- Maximum ultimate load occurs due to point load acting on the shear centre the percent of increasing about (35.84%).
- There are two types of global initial geometric imperfections may exist in mono-symmetric cold-formed steel channel section beams, namely negative and positive imperfections. The negative imperfection is identified by the lateral movement of the cross-section outward with anticlockwise twist while the cross-section deformed in ward with a clockwise twist on the case of positive imperfection.
- The yield stress of the material is increased in corners will reach the fully plastic stress compared with flatting edges is lower, From the results the convergence between two cases about (59.42%) and cold forming is known to increase the yield and ultimate strength of the material due to cold working or strain hardening.
- Higher ultimate load capacities were achieved in the higher thickness and t_w is not equal to t_f for the same area of cross section under the same conditions of loading.
- Cold-formed channel beam the lateral torsional buckling moment occur in between elastic and inelastic critical moment . The convergence about (97.83%) with the experimental values and (89.78%) numerical equations.
- Due to differences in the degree of cold working in the flat parts and the corner regions, the mechanical properties vary over the cross-section.

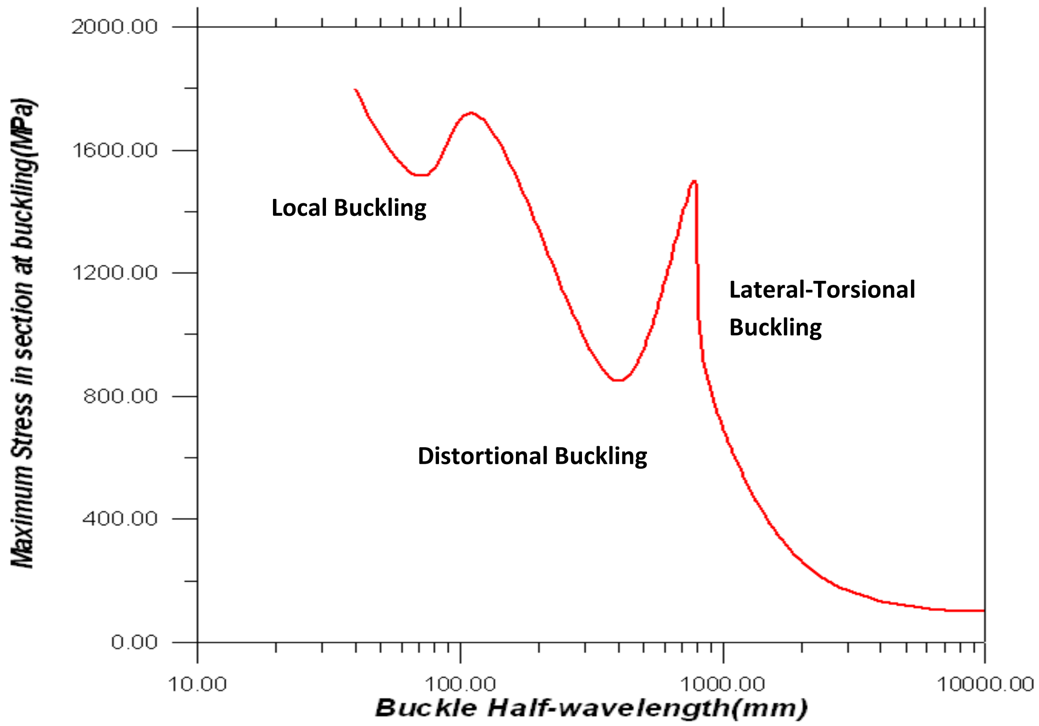


Fig.1 Buckling modes of a cold-formed steel Channel Section [Kankanamge2010]

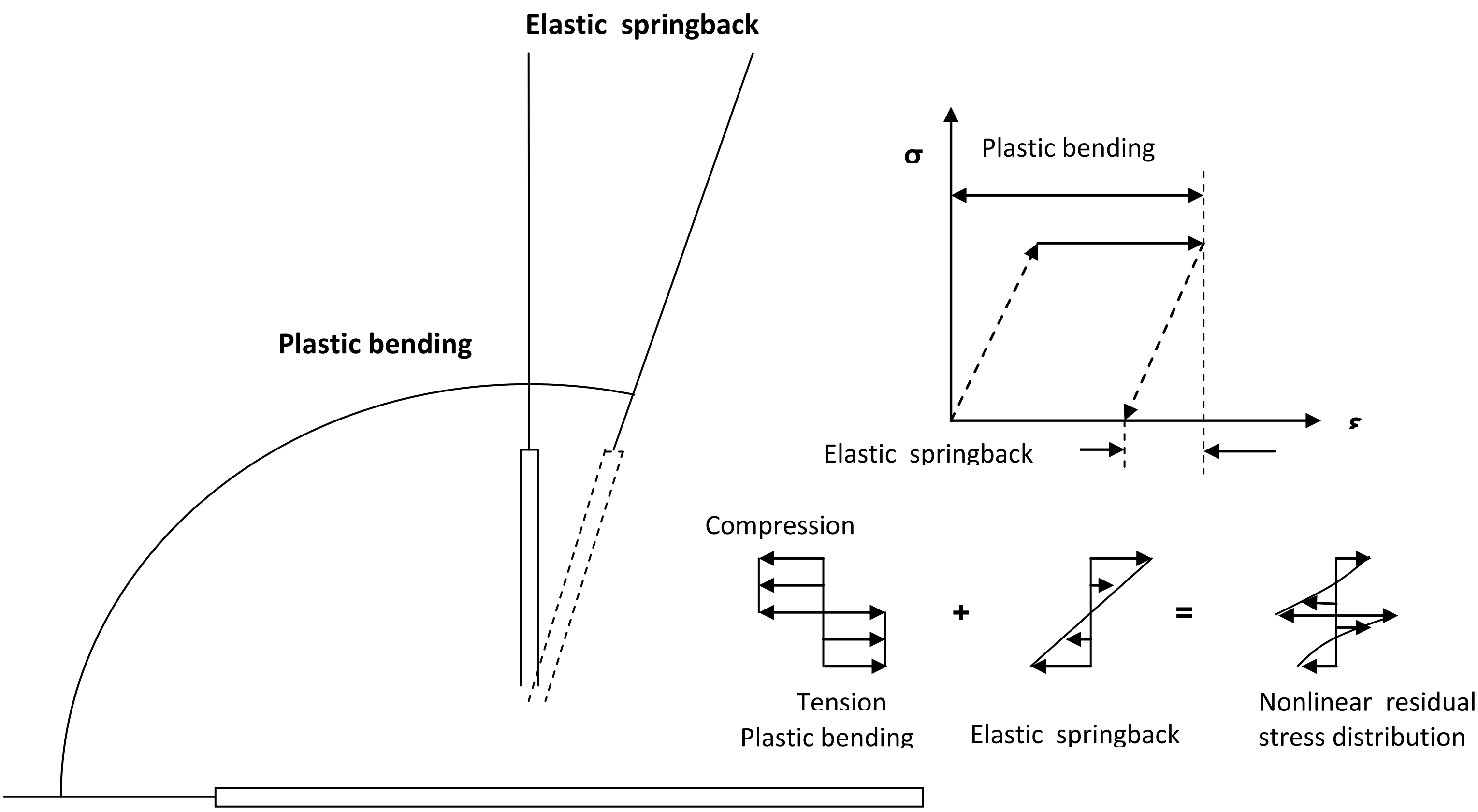
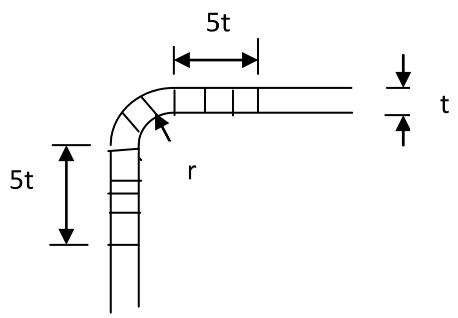


Fig.2 Forming a bend: plastic bending and elastic springback of thin sheets results in a nonlinear through-thickness residual stress distribution[Cristopher and Schafer 2009]

Table 1 Conditions for welding cold-formed zones and adjacent material.

r/t	Strain due to cold forming(%)	Maximum thickness(mm)		
		Generally		Fully killed Aluminum-killed steel (Al≥.02%)
		Predominantly static loading	Where fatigue predominates	
≥25	≥2	Any	Any	Any
≥10	≥5	Any	16	Any
≥3.0	≥14	24	12	24
≥2.0	≥20	12	10	12
≥1.5	≥25	8	8	10
≥1.0	≥33	4	4	0



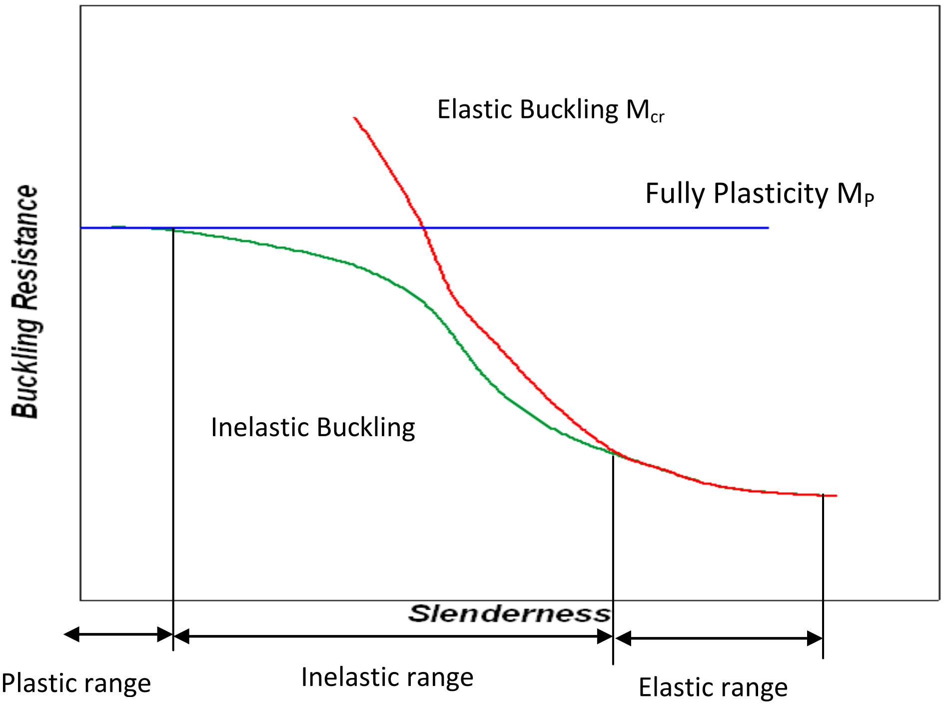


Fig.(3) Effect of slenderness on the Buckling resistance of beams [Real

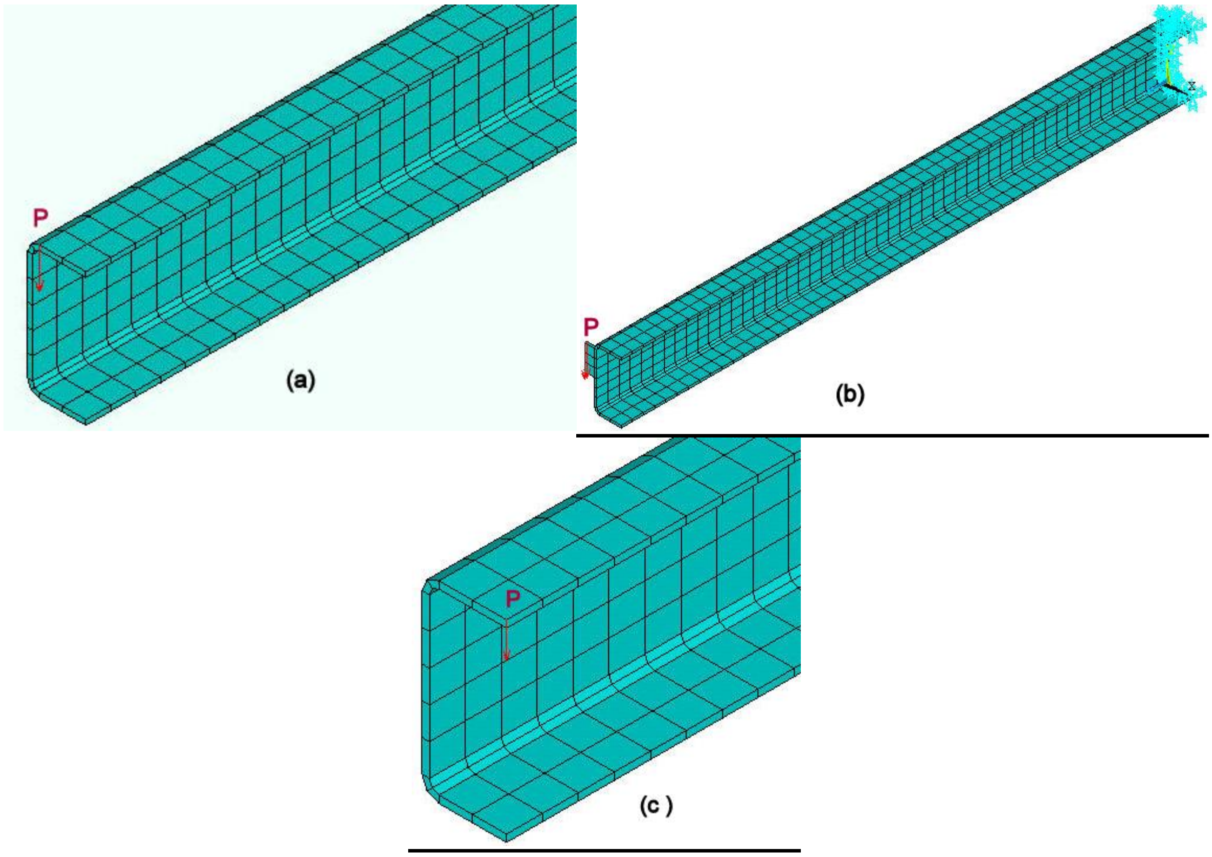
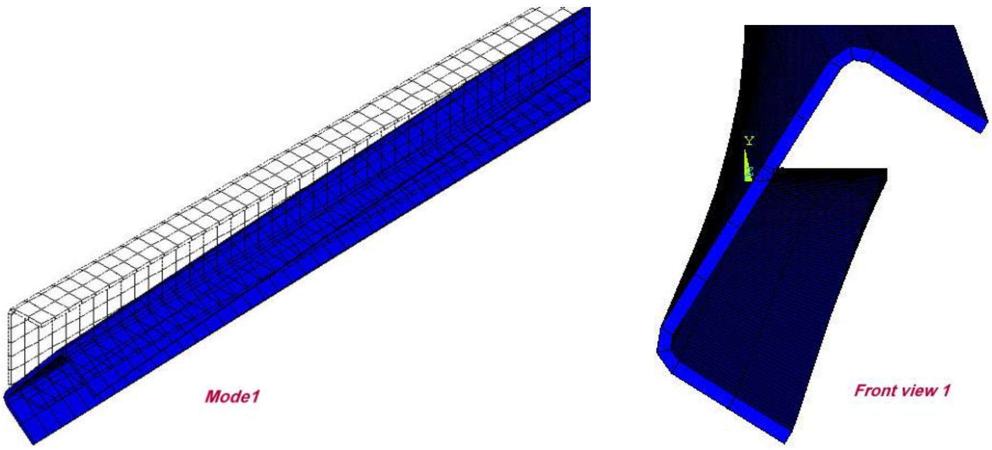


Fig. 4 Finite element model of channel beam a) P acts on the rounded edge b) P acts on the shear center c) P acts on the edge of flange.



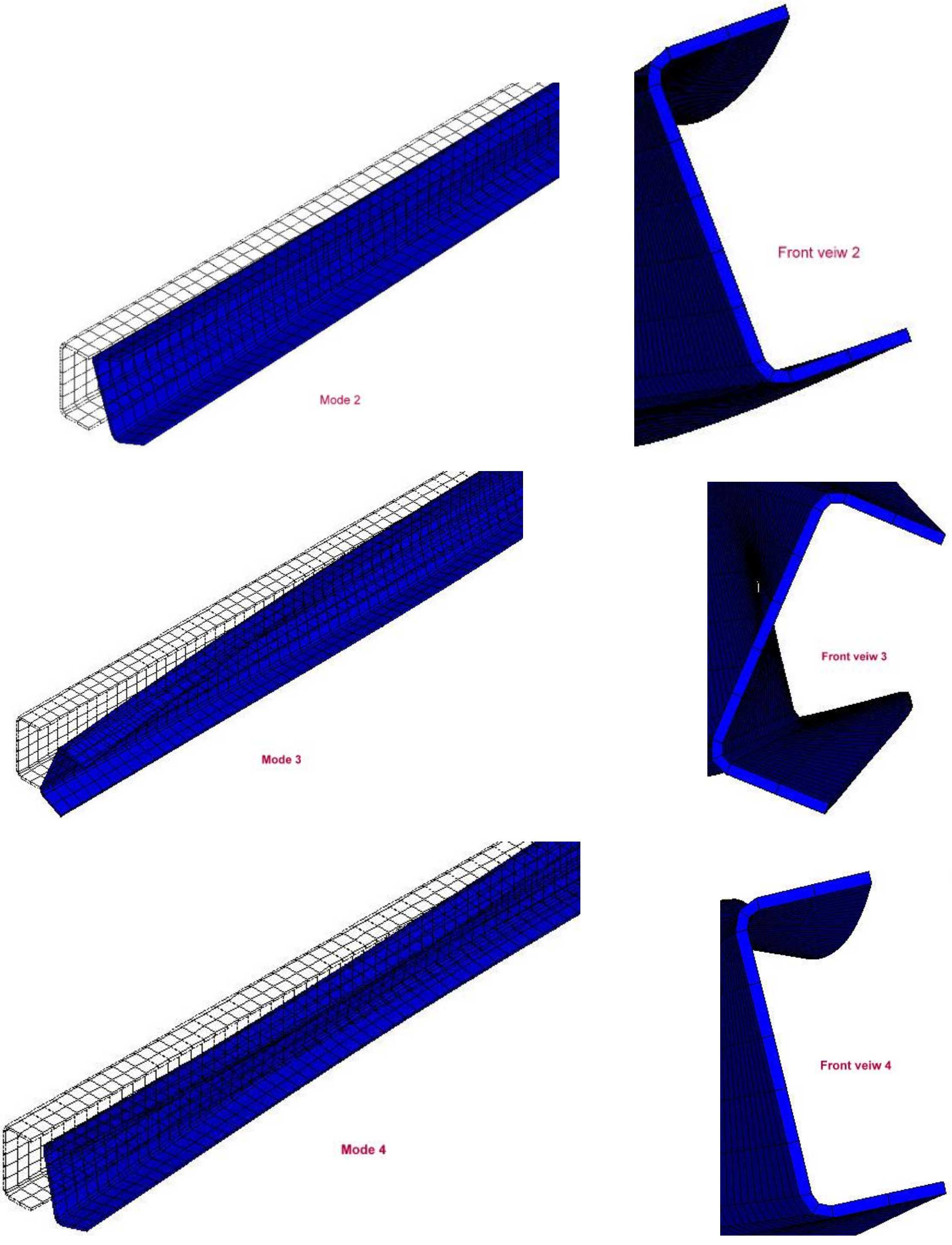


Fig. 5 Deformed shape due to lateral-torsional buckling for case (1) of beam loading

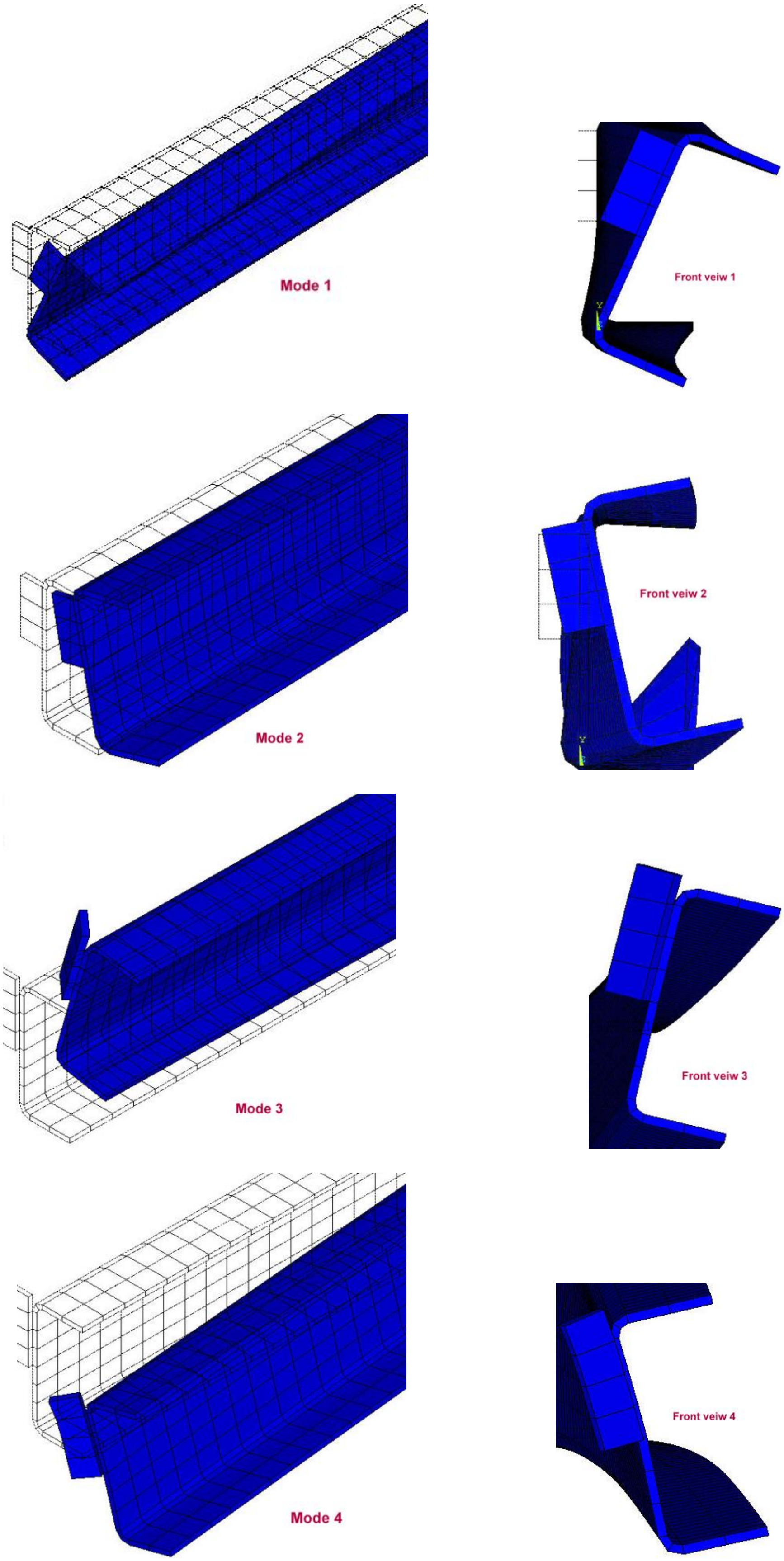


Fig. 6 Deformed shape due to lateral-torsional buckling for case (2) of beam loading

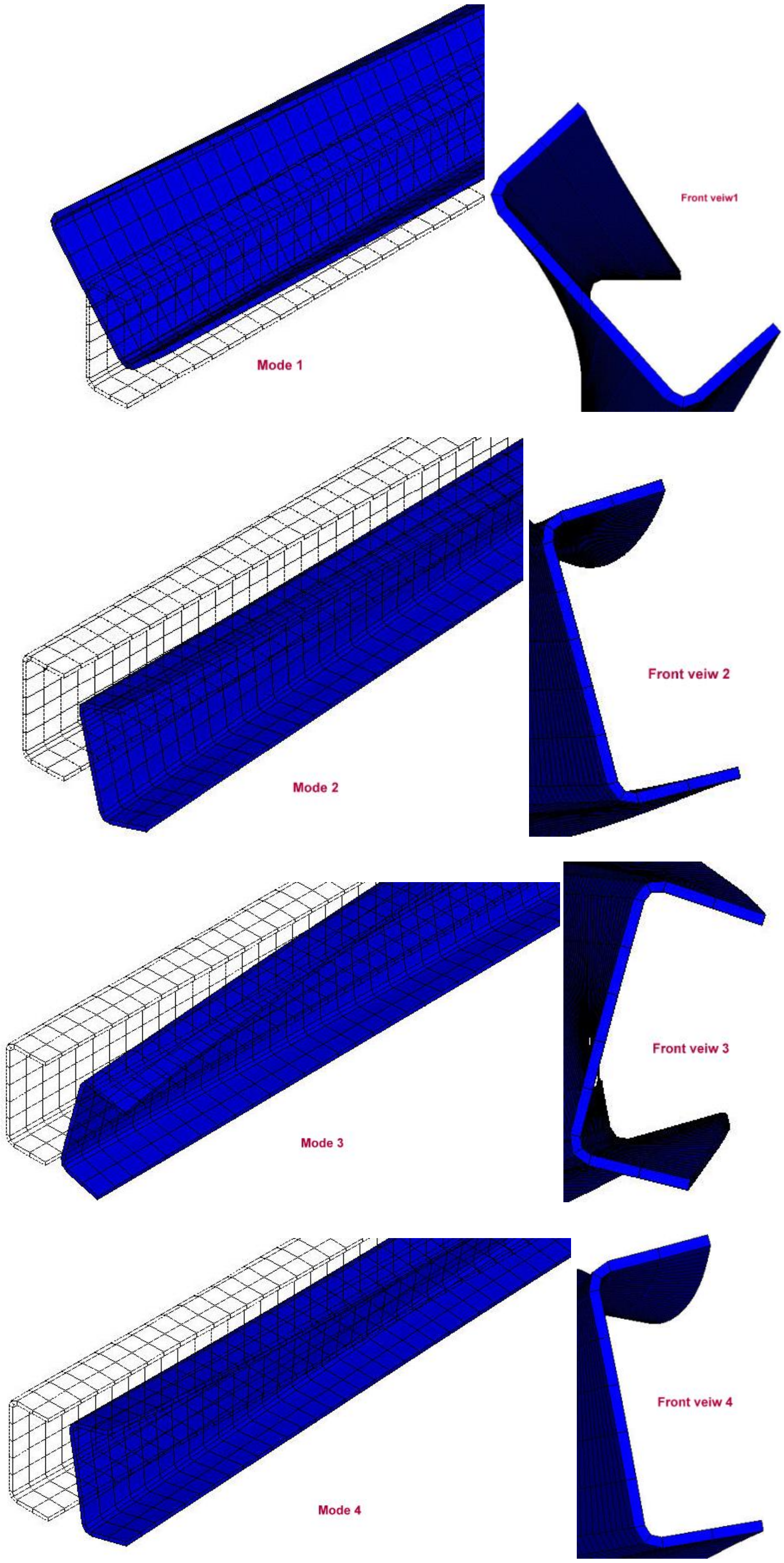
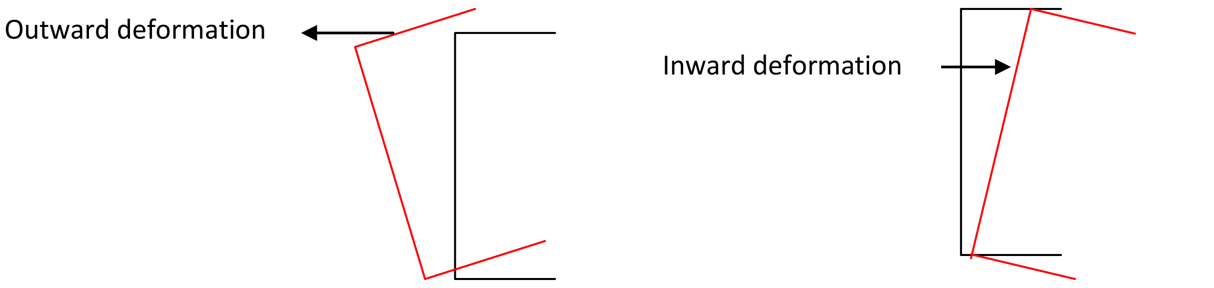
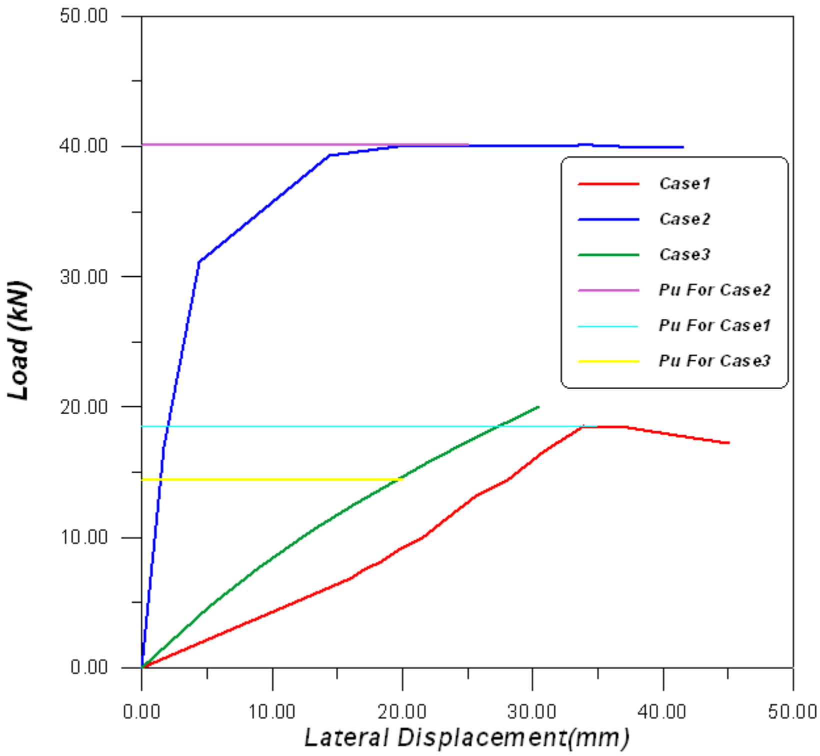


Fig. 7 Deformed shape due to lateral-torsional buckling for case (3) of beam loading

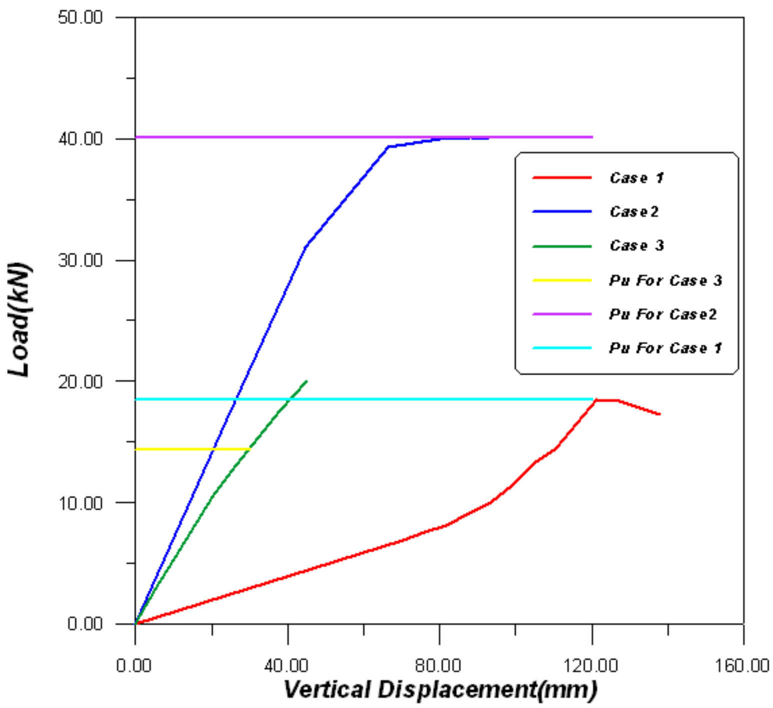


(a) Negative Imperfection (b) Positive Imperfection

Fig. 8 Negative and Positive Geometric

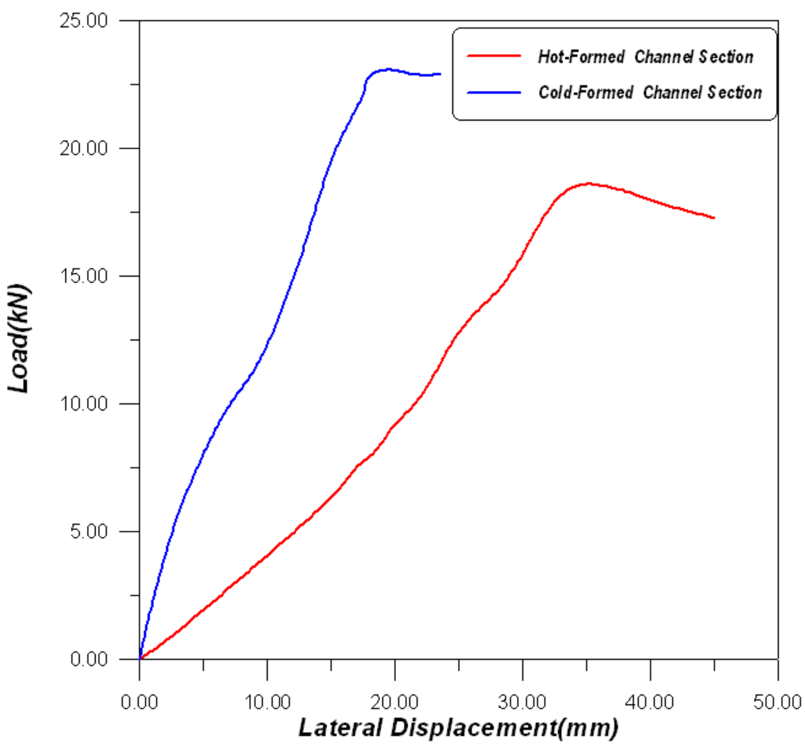


(a)

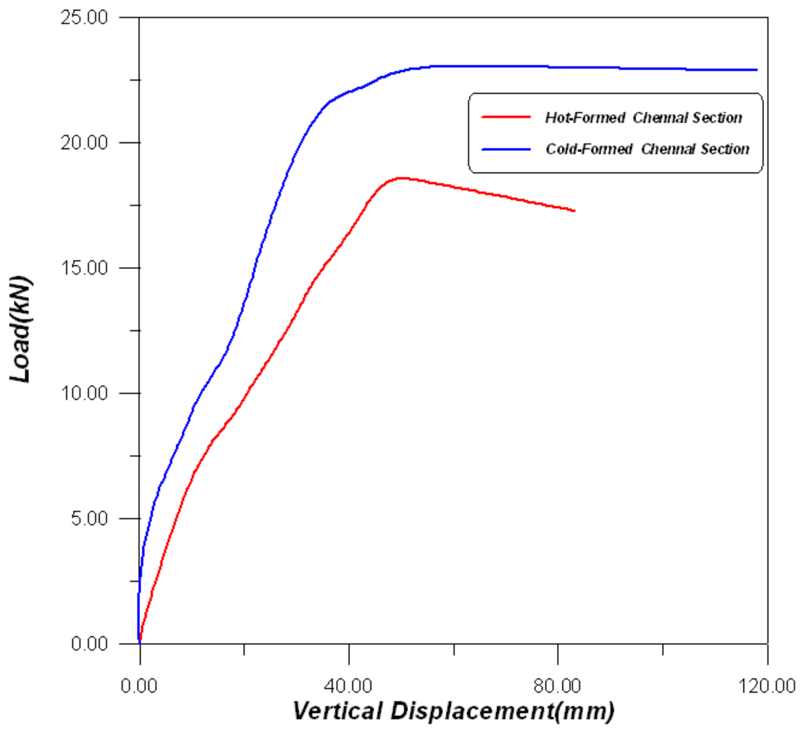


(b)

Fig.(9) Load versus deflection plots a) Lateral deflection b) Vertical

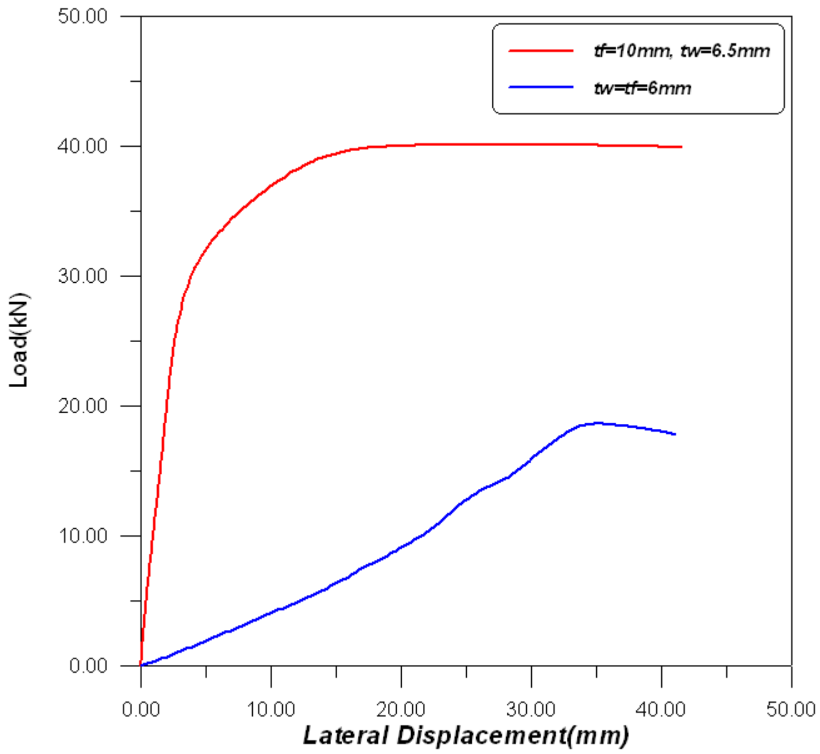


(a)

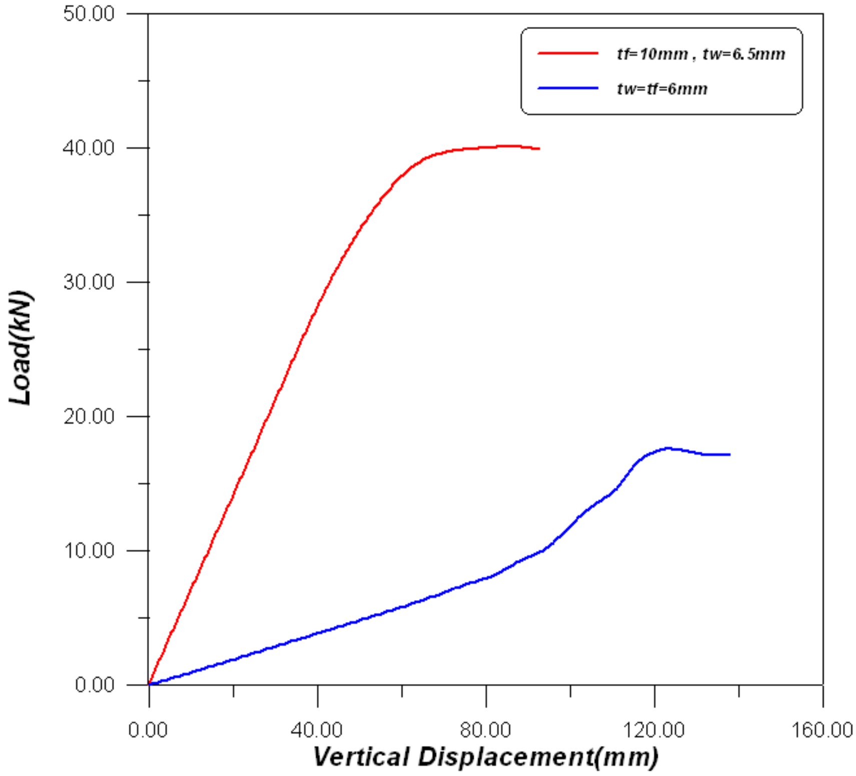


(b)

Fig. 10 Load versus deflection plots a) Lateral deflection b) Vertical Deflection

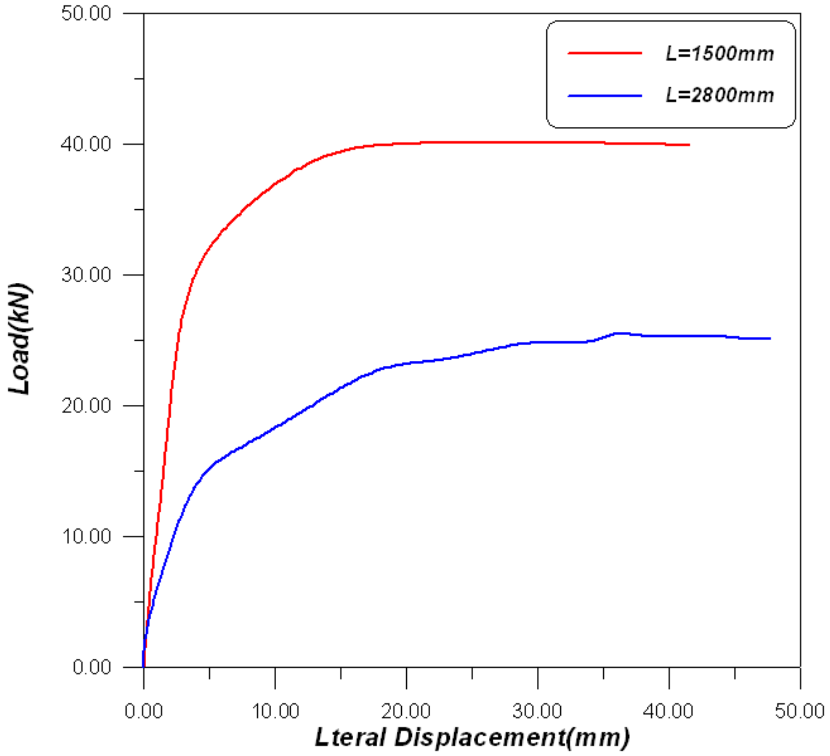


(a)

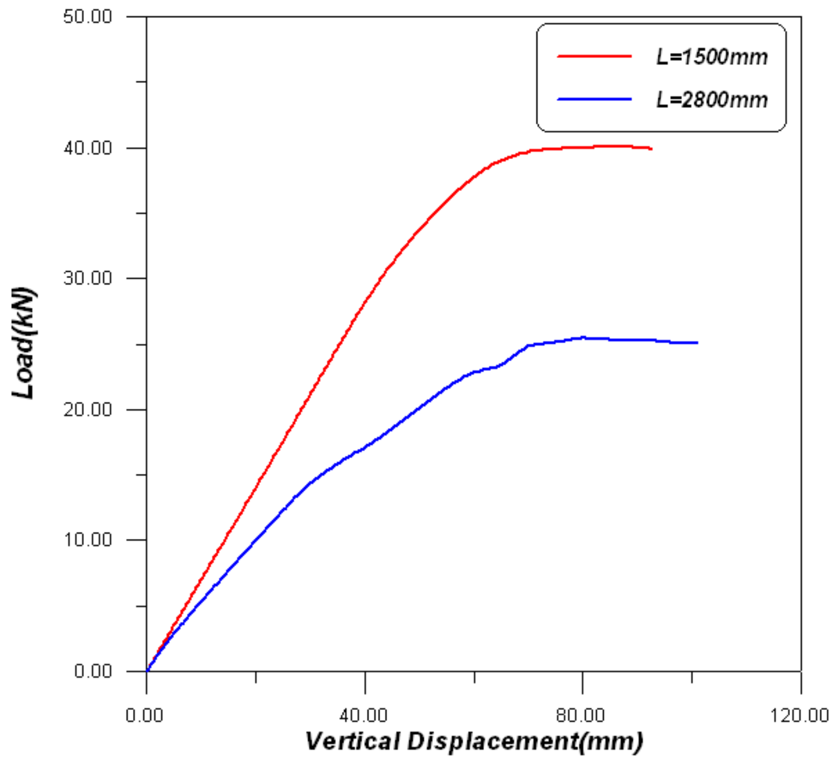


(b)

Fig. 11 Load versus deflection plots a) Lateral deflection b) Vertical deflection



(a)



(b)

Fig. 12 Load versus deflection plots a) Lateral deflection b) Vertical deflection

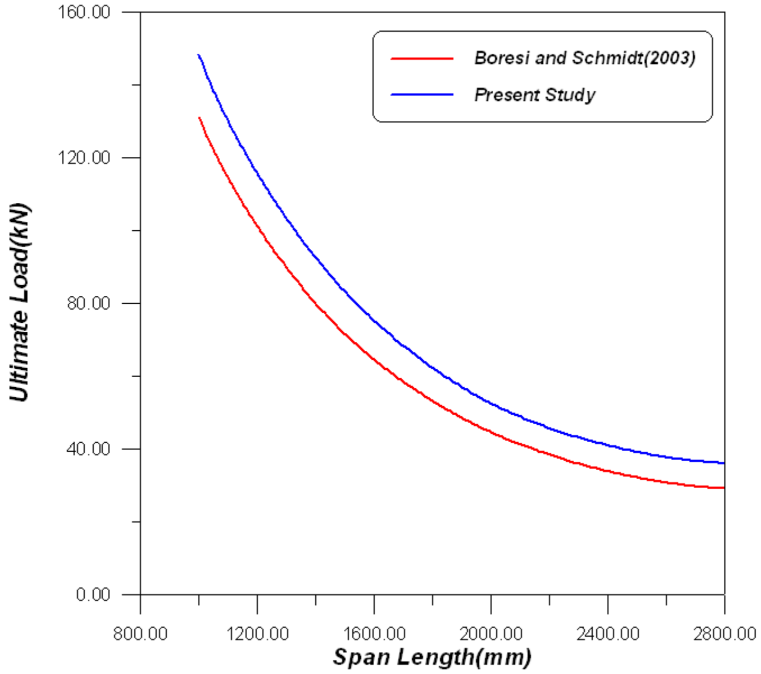


Fig. 13 Effect of span length for cantilever beam on the ultimate load capacity of cold formed channel beam.

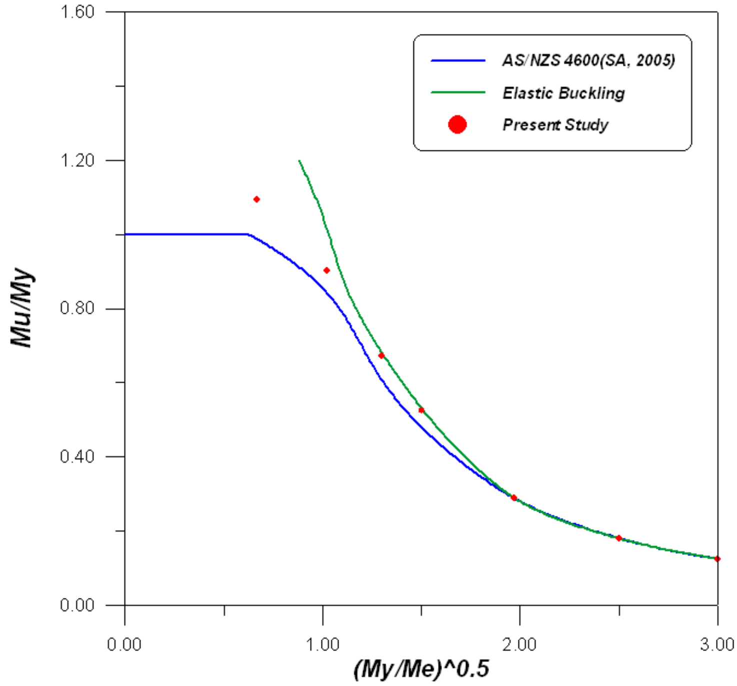


Fig. 14 Comparison between FEA [ANSYS] with AS/NZS 4600

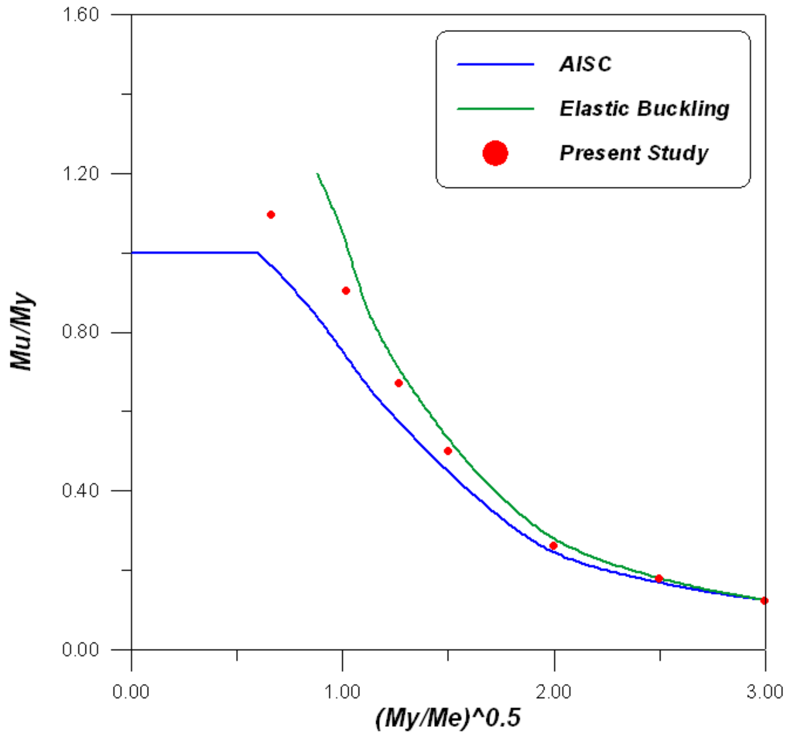
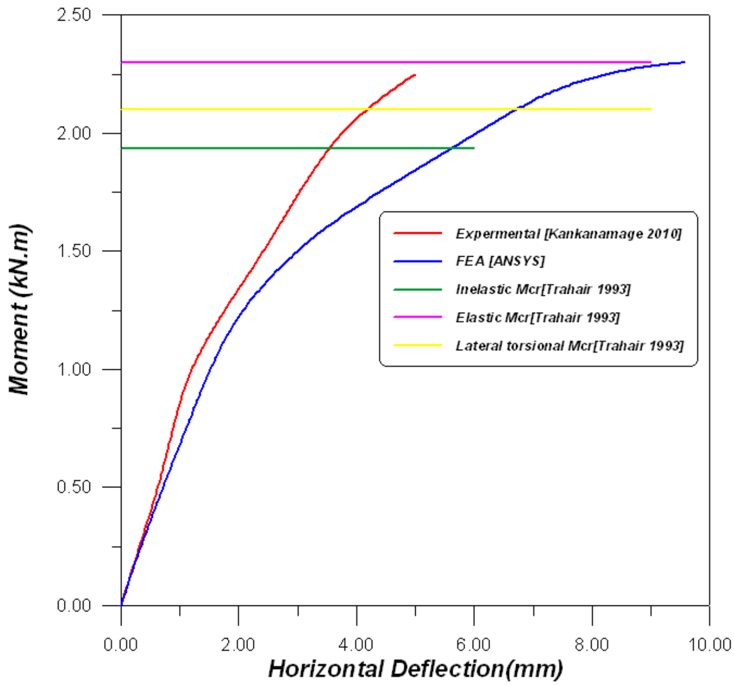
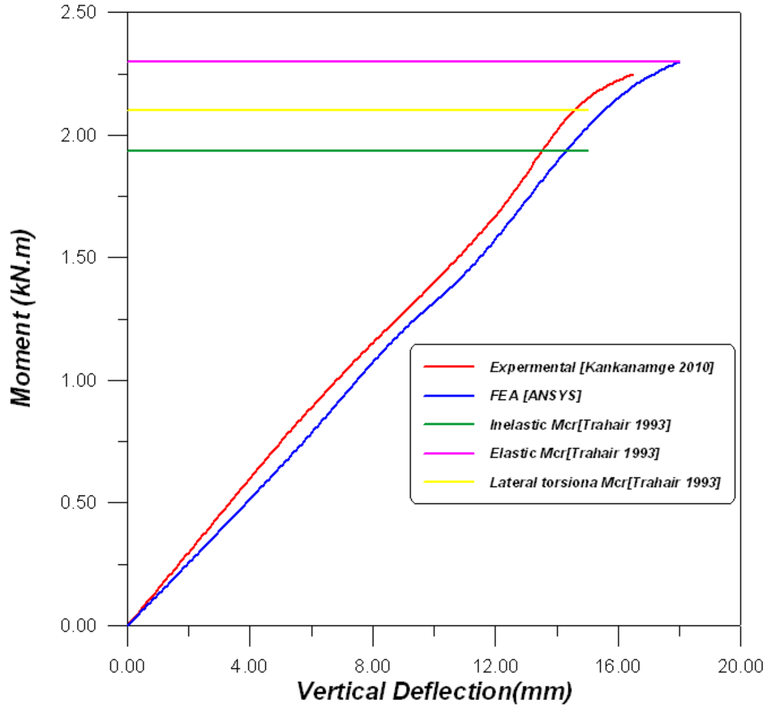


Fig. 15 Comparison between FEA [ANSYS] with

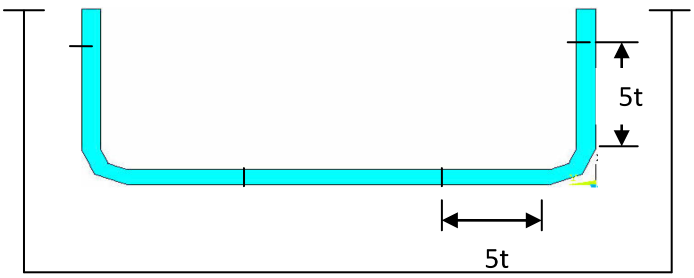


(a)



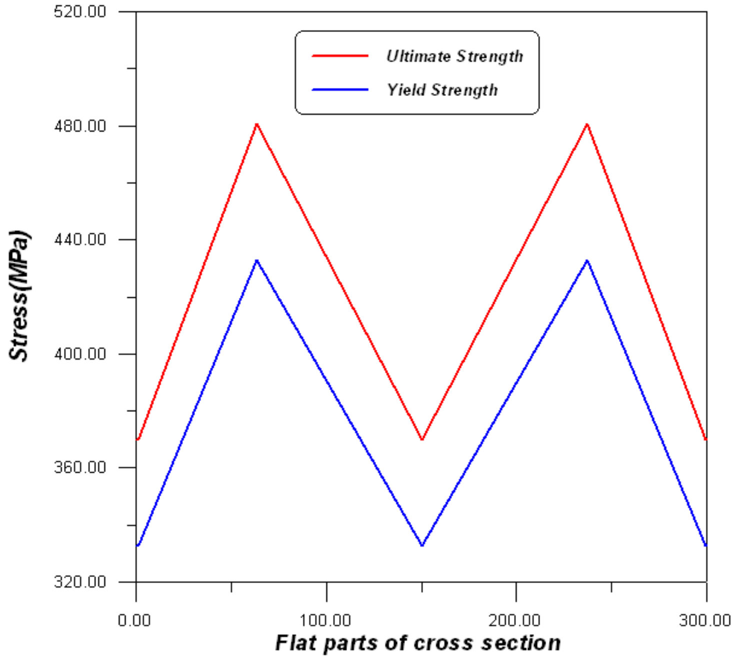
(b)

Fig. 16 Comparison between FEA **[ANSYS]** with Experimental work **[Kankanamge 2010]** and **[Trahair 1993].**



Flat part of cross section =$2b_f+h_w+2r$

(a)



(b)

Fig. 17 Effect of cold work on the mechanical properties of cold-formed steel channel section

a) steel channel section b) stresses on flat parts of cross section

REFRENCES

AISC-Manual , Thirteen Edition LFRD and ASD, According to AISC-Specifications 2005.

Chen ,J. ,and Young, B., "Corner properties of cold-formed steel sections at elevated temperatures" Thin-walled Structures 2006,44(2):216-223.

Chu,X., -t, Kehle, R and Li, L., -y, "Lateral torsion buckling analysis of partial laterally restrained thin-walled channel section beams", Journal of Constructional Steel Research 2004. 60(8): 1159-1175.

Cristopher D. Moen andB.W. Schafer "DIRECT STRENGTH DESIGN OF COLD-FORMED STEEL MEMBERS WITH PERFORATIONS The Johns Hopkins University March 2009.

Eurocode 3(2006), ENV, 1993-1-1:1992/ A1:1994, Design of steel structures General Rules and Rules for building, Brussels.

Hill, H., "Lateral buckling of channels and Z-beams", Trans. ASCE 1954, 119:829-841.

Kankanamge, N. D.," Structural behavior and design of cold-formed steel beams at elevated temperatures", Ph.D., Queensland University of Technology in Partial, May, 2010.

Key, P. and Hancock, G., "A theoretical investigation of the column behavior of cold-formed shape squre hollow sections" Thin-Walled Structures, 1993,16(1):3164.

Pi, Y. and Trahair, N. S., "Pre-buckling deflections and lateral buckling II: Theory, Journal of Structural engineering 1992b,118(11):2967-2985.

Pi, Y. and Trahair, N. S., "Pre-buckling deflections and lateral buckling I: Theory, Journal of Structural engineering 1992a,118(11):2949-2966.

Pi, Y., Put, B.M. and Trahair, N. S., " lateral buckling strengths of cold-formed channel sections beams", Research Report No. R748, The University of Sydney, Australia, 1997.

Products hand book" Structural steel" , Continental steel PTE, LTD, Singapore 629586.

Quach, W.M., Teng, J.G., and Chung, K.F. (2004). "Residual stresses in steel sheets due tocoiling and uncoiling: a closed-form analytical solution." Engineering Structures, 26, 1249-1259.

Quach, W.M., Teng, J.G., and Chung, K.F. (2006). "Finite element predictions of residual

Real, PMM. Czelir, R., Simoesdasilva, L., Santiago, A. and Piloto, P. " The effect of residual stresses in the lateral-torsional buckling of steel-I beams at elevated temperatures" Joural of Constructional Steel Research, 2004 a , 60(3-5), 783-793.

Schafer, B. W. [2002], "Local distortional, and Euler buckling of thin-walled column", Journal of Structural Engineering March. , 2002.

Schafer, B.W., and Ádàny, S. (2006). "Buckling analysis of cold-formed steel members using CUFSM: conventional and constrained finite strip methods." Eighteenth International Specialty Conference on Cold-Formed Steel Structures, Orlando, FL.

Seah, L. K. and Khong, P. W., "Lateral-torsional buckling of channels beams" , J., Construct Steel Research 17(1990),265-282.

Shanley, F.R. (1957). Strength of Materials, McGraw-Hill Book Company, New York, NY.

Standards Australia/Newzeland, **AS/NZC, 4600"** Cold-formed steel structures ", SydneyAustralia,285.stresses in press-braked thin-walled steel sections." Engineering Structur es, 28, 160- 619.

Trahair, N. S. and Bradford M., "The behavior and design of steel structures. "Chapman and Hall NewYourk,1988.

Trahair, N. S., "Flexural torsional buckling of structures" ,Talyor and Francis Group,1993.

US Steel. (1985). The Making, Shaping, and Treating of Steel, 10th Edition, Herbick & eld,Pittsburgh, PA.

Yu,. W. W., "Cold-formed steel design",3rd Edition, John Wiley and Sons, Inc., USA.2000.