The Iraqi Journal For Mechanical And Material Engineering, Vol.19, No3,Sept. 2019

W1FINITE ELEMENT ANALYSIS OF PIPES CONVEYING FLUID MOUNTED ON VISCOELASTIC FOUNDATIONS

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Abstract

In this study, the stability of a simply supported pipeline conveying fluid with different velocities and resting on viscoelastic foundation is investigated by using finite element analysis, and the critical fluid velocity with different parameters such as stiffness and viscous coefficients of foundation are obtained. This structural system could be found in pipes conveying petrol, water, and sewage. The foundation is simulated using the modified Winkler's model to introduce the effect of time dependent viscosity term. Some known results are confirmed and some new ones obtained. Two components of foundation, stiffness and viscosity, seemed to act on the critical flow velocity of the pipe in contrary trend. Where, increasing the foundation stiffness tended to increase the critical flow velocity in the pipe. While, increasing foundation viscosity attempted to decrease it. At some ranges of pipe length, the foundation viscosity effect seems to be more extreme. Increasing the fluid velocity leads to monotonic reduction in the system damping ratio. Two parameters, pipe length and fluid density which relate to the natural frequency of pipeline conveying fluid are studied in detail and the results indicate that the effect of Coriolis force on natural frequency is become more effective by increasing pipe length and fluid density besides increasing fluid flow velocity.

Keywords: Finite element; Fluid- structural interaction; Viscoelastic foundation; Modified Winkler's model; Pipe**s;** Stability.

LIST OF SYMBOLS

Symbol	Definition	Units
A	Cross-sectional flow area	m^2
b	Width of the beam in contact with the base foundation	m
$\hat{\mathbf{C}}_1$	Foundation viscous matrix	-
$\hat{\mathbf{C}}_2$	Damping matrix caused by Coriolis force	-
$\boldsymbol{e}$	Element	-
E	Modulus of elasticity of pipe	N/m^2
$f(x,t)$	Intensity of reaction force of foundation	N/m^2
F	Reaction force inside the pipe	N
g	Acceleration constant	m/s^2
H	Characteristic matrix	-
I	Pipe second moment of area	m_4
II	Unity matrix	-
k_o	Foundation stiffness coefficient per unit length	N/m^2
k_v	Foundation stiffness coefficient per unit area	N/m^3
$\hat{\mathbf{k}}_1$	Stiffness matrix of pipe	-
$\hat{\mathbf{k}}_2$	Foundation stiffness matrix	-
$\hat{\mathbf{k}}_3$	Stiffness matrix comes from flow around deflected pipe	-
L	Length of the pipe	m

Received : 2011 - 4 - 21
Accepted : 2011 - 5 - 15

l	Element length of pipe	m
M	Fluid mass per unit length	kg/m
m	Pipe mass per unit length	kg/m
$\hat{\mathbf{m}}$	Pipe mass matrix	-
$\breve{M}$	Bending moment	N/m
$\mathbf{N_i}$	Shape function	-
p	Pressure inside the pipe	N/m^2
Q	Shear force	N
$\breve{q}$	Wall shear stress	N/m^2
$\mathbf{q}$	Lateral displacement of pipe	m
$\dot{\mathbf{q}}$	Lateral velocity of pipe	m/s
$\ddot{\mathbf{q}}$	Lateral acceleration of pipe	m/s^2
S	Pipe inner perimeter	m
T	Tension force in the pipe	N
t	Time	s
U	Fluid velocity relative to the pipe	m/s
x,W	Cartesian axes	-
μ	Foundation damping coefficient per unit length	$N.s/m^2$
μ_v	Foundation stiffness coefficient per unit area	$N.s/m^3$
λ	Eigen values	-
ζ	Damping ratio	-

INTRODUCTION

Piping systems are widely utilized to convey fluids in many industrial fields, ranging from chemical plants to biological engineering systems. Examples include fuel pipes in engine systems, heat transfer pipes in power generation plants, refrigerators, air-conditioners, heat exchangers, chemical plants piping, hydropower systems and so forth. Piping vibration problems are therefore very important in industry. The instability problem of flexible pipes conveying fluid provides a paradigm for the modeling and analysis of the instability mechanisms of fluid-structure interaction systems. The stability and dynamic characteristics are now well understood. The dynamics are known to be sensitively dependent on flow velocity and support/boundary conditions. In general, it has been established that an initially straight pipe that conveys a fluid with a relatively low speed is stable. In other words, each disturbance applied to that pipe causes a vibration that decreases with time. It has been also found that for fluid speed values higher than a certain value (the critical flow velocity) even a small disturbance could result in a system vibration that increases with time. In latter circumstances, therefore, the system equilibrium state is referred as unstable. The first serious study of the dynamics of pipes conveying fluid is due to Bourrieres, [1939], who derived the correct equations of motion and carried its analysis remarkably far, reaching admirably accurate conclusions regarding stability, in particular concerning the cantilevered system. Major research on the regarding stability and vibration of flexible pipes conveying fluid started in the 1950s in relation to the design of pipelines conveying oil. Lottati and Kornecki ,[1986], found that the critical flow velocity of a fluid conveying pipe on Winkler foundation is higher than the critical flow velocity of that pipe without foundation. In this manner, the Winkler foundation is proved to have a stabilizing effect on the pipe. Chen ,[1991], used the Vlasov foundation model to describe the vibration of a pipeline containing flowing fluid and supported on an elastic foundation. The critical flowing velocity of such pipe was solved analytically using interaction method .He concluded that elastic support would reduce that amplitude of the pipeline vibration. For pipes of a finite length, the dynamical behavior depends strongly on the type of boundary conditions at both ends (Lee and Mote, [1997]). Elishakoff and Impolonia, [2001] and Djondjorov , [2001], have studied the dynamic stability of cantilevered pipes on foundations of constant modulus that support only a part of the pipe span. They have found that such foundations could either destabilize or stabilize the pipe depending on the position and length of the foundations. Djondjorov, Vassilev and Dzhupanov, [2001], and Djondjorov, [2001], have examined cantilevered pipes on Winkler foundations whose modulus is a certain sixth-, second- or first-order polynomial. They have concluded that all such foundations stabilize the pipe. Païdoussis, [2004], studied numerically pinned-clamped and clamped-pinned pipes conveying fluid. He found that to predict the dynamical behavior of the clamped-pinned pipe, even 8 significant-figure accuracy was not good enough. The imaginary part of the complex Eigen frequency seemed to be negative, implying unstable behavior for any flow velocity greater than zero. Lumijärvi, [2006], studied the optimal design of cantilevered fluid-conveying pipes. The aim of his study was to maximize the critical flow speed of the fluid by means of additional masses, supporting springs or dampers along the length of the pipe. The optimization problem was formulated by modeling the pipe by finite element method, using Euler-Bernoulli beam elements. The locations of the additional masses, springs and dampers and the properties of these elements (mass, spring constant and damping constant) were chosen as design parameters. The maximization problem for the critical fluid flow speed was solved by the sequential quadratic programming technique. Huang et al, [2010], applied the eliminated element-Galerkin method to calculate the natural frequency with different boundary conditions based on typical transverse vibration model. Then the relationship between simplified natural frequency of the pipeline and that of Euler beam was discussed. In a given boundary condition, the four components (mass, stiffness, length and flow velocity) which relate to the natural frequency of pipeline conveying fluid were studied in detail and the results indicate that the effect of Coriolis force on natural frequency was inappreciable. Mahrenholtz, [2010] ,extended the Winkler's model to account the effect of time dependent in the simplest case to make it viscous. He applied the viscous model to solve a problem of rotating wheel sets on polymer rubber sheet. Good agreements between the proposed model and experimental data were obtained. Thomsen and Dahl, [2010], investigated the resonant vibrations of a fluid-conveying pipe, with special consideration to axial shifts in vibration phase accompanying fluid flow and various imperfections. Small imperfections related to elastic and dissipative support conditions were specifically addressed, but the suggested approach was readily applicable to other kinds of imperfection, e.g. non-uniform stiffness or mass, non-proportional damping, weak nonlinearity, and flow pulsation. Rinaldi et al,[2010], investigated the effects of flow velocity on damping, stability, and frequency shift of microscale pipes containing internal fluid flow. The analysis was conducted within the context of classical continuum mechanics, and the effects of structural dissipation (including thermo elastic damping in hollow beams), boundary conditions, geometry, and flow velocity on vibrations were discussed. The study showed that flow-induced damping and frequency shifts in representative single-crystal silicon structures could exceed the typical specifications for resonant micro sensors. To the best of our knowledge, other studies on dynamic stability of pipes on variable elastic foundations are not reported in the literature. It can be found in Païdoussis, [1998].

From the review of literature, it is found that the study of flow induced vibration in pipes conveying fluid mounted on viscoelastic foundation has not yet been explored so far. The aim of this is to clarify whether the critical flow velocity depends on the magnitude of the foundation stiffness, foundation damping, the pipe length, pipe thickness and fluid density for a simply supported pipe conveying fluid.

VISCOELASTIC EXTANSION OF WINKLER FUNCTION

An analysis of the bending of beams on a viscoelastic foundation, if based on the Winkler model, is derived from the assumption that the foundation's reaction forces are proportional at every point to the deflection of the beam at that point. The differential equation of the elastic line is based on the assumption that a straight beam is supported along its entire length by a viscoelastic medium and subjected to vertical forces acting in the principal plane of the symmetrical cross section (see Figure 1). Under these conditions, the beam will deflect, thus producing continuously distributed reaction forces in the supporting medium. One may make the fundamental assumption that the intensity (f) of the reaction forces at any point is proportional to the deflection of the beam $W(x,t)$ at that point. The reaction forces are assumed to act vertically and in opposition to the deflection of the beam. Hence, where the deflection is directed downward (in a positive direction) the supporting medium will be compressed. However, where the deflection is negative, tension is produced; for the purposes of this research, the supporting medium is assumed able to take up such tensile forces. If a beam has a uniform cross section and b is its constant width, then a unit of deflection of this beam will cause reaction in the foundation; consequently, at a point where the deflection is $W(x,t)$, the intensity of the distributed reaction, per unit length of the beam, will be (Mahrenholtz, [2010];

$$p(x,t) = k_o W + \mu \frac{dW}{dt} \tag{1}$$

Where k_o and μ are the stiffness and damping (viscous) coefficients of foundation per unit length respectively. The assumption $f(x,t)$ implies that the supporting medium is viscoelastic. Its material, then, acts in accordance with Kelvin-Voigt model. Its viscoelasticity, therefore, may be characterized by the force which, distributed over a unit area, will cause a deflection equal to that unit. The constant values of the supporting medium, k_v and μ_v, are called the moduli of stiffness and viscous foundation respectively. Where,

$$k_o = bk_v \tag{2}$$

$$\mu = b\,\mu_v \tag{3}$$

The units of the moduli k_v and μ_v are in (N/m^3) and (N s/m^3) respectively. While (b) is the width of the beam in contact with the base foundation. However, it should be remembered that k_o and μ includes the effect of the width of the beam and will be numerically equal to only if the beam is of a unit width.

DERIVATION OF GOVERNING DIFFERENTIAL EQUATION

The detailed analysis of the dynamics of straight flexible pipes conveying fluid is described by Païdoussis, [1998] and Païdoussis, [2004]. In this section, the modeling and calculation method based on these papers are introduced. When a pipeline rests on a viscoelastic medium such as polymers, a model of the viscoelastic medium must be included in the governing differential equation. The physical system analyzed is shown in Figure(2-a). Forces and moments acting on the fluid and pipe elements, respectively, are shown in Figure (2-b and c). The pipe is considered to be slender, and its lateral motions, $W(x,t)$, to be small and of long wavelength compared to the diameter. The system consists of a uniform pipe of length (L) , pipe mass per unit length (m) , flexural rigidity (EI), conveying fluid of mass per unit length (M), flowing axially with velocity (U) , and mounted on viscoelastic foundation with stiffness (k_o) and viscous damping (μ). The cross-sectional flow area is (A), inner perimeter is (S) and the fluid pressure is (p). Consider then elements δx of the fluid and the pipe, as shown in Figure(2-b and c). The fluid element of Figure (2-b) is subjected to: (i) pressure forces, where the pressure $p= p(x,t)$ because of frictional losses, p is measured above the ambient pressure, and t is the time; (ii) reaction forces of the pipe on the fluid normal to the fluid element, $F\delta x$, and tangential to it, ($\breve{q}\ S\ \delta x$), associated with the wall-shear stress $\breve{q}$; (iii) gravity forces ($M\ g\ \delta x$) in the W- direction. Balancing the forces in W-direction of the fluid element while keeping in mind the small deflection approximation, yields

$$-F - M\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 W = pA\frac{\partial^2 W}{\partial x^2} \tag{4}$$

The sum of forces parallel to the pipes axis for constant flow velocity gives

$$A\frac{\partial p}{\partial x} + \breve{q}S = 0 \tag{5}$$

Similarly, for pipe element of Figure (2-c) one obtains

$$\frac{\partial T}{\partial x} + \breve{q}S = 0 \tag{6}$$

And the forces normal to the pipe axis for small deformation

$$\frac{\partial Q}{\partial x} + F + T\frac{\partial^2 W}{\partial x^2} + k_o W + \mu\frac{\partial W}{\partial t} = m\frac{\partial^2 W}{\partial t^2} \tag{7}$$

where

$$Q = -\frac{\partial \tilde{M}}{\partial x} = -EI\frac{\partial^3 W}{\partial x^3} \tag{8}$$

From eq.(5) and eq.(6), the wall shear stress $\breve{q}$ is eliminated to result in

$$\frac{\partial(pA-T)}{\partial x} = 0 \tag{9}$$

The pipe end where $x=L$, the tension T in the pipe is zero and the fluid pressure is equal to ambient pressure, thus $p=T=0$ at $x=L$,

$$pA - T = 0 \tag{10}$$

Combining all the above equations yields the following governing equation

$$EI\frac{\partial^4 W}{\partial x^4} + k_o W + \mu\frac{\partial W}{\partial t} + MU^2\frac{\partial^2 W}{\partial x^2} + 2MU\frac{\partial^2 W}{\partial x \partial t} + (m+M)\frac{\partial^2 W}{\partial t^2} = 0 \tag{11}$$

The term ($EI\frac{\partial^4 W}{\partial x^4}$) represent a force component acting on the pipe because of pipe bending. The terms ($k_o W$) , ($\mu\frac{\partial W}{\partial t}$) represent the force component acting on the pipe that comes from foundation stiffness and foundation damping respectively. The expression ($MU^2\frac{\partial^2 W}{\partial x^2}$) represent the force component acting on the pipe as a result of flow around a deflected pipe (curvature in pipe). The term ($2MU\frac{\partial^2 W}{\partial x \partial t}$) is the inertial force associated with the Coriolis acceleration arising because the fluid flows with velocity U relative to the pipe. In addition, this expression is the so-called anti-symmetric whirligig “damping” item. Because of its effect, the fluid structural interaction model belongs to complex eigenvalue problem. While the expression ($(m+M)\frac{\partial^2 W}{\partial t^2}$) is force acting on the pipe because of inertia of the pipe and the fluid flowing through it. A remarkable feature of eq. (11) is the total absence of fluid-frictional effects, which at first sight might appear to be an idealization. However, within the context of the other approximations implicit in this linearized equation, it may rigorously be demonstrated that fluid-frictional effects play no role in the dynamics of the system, a fact first shown by Benjamin, [1961 parts a and b].

FINITE ELEMENT DISCRETIZATION

Eq. (11) is a binary partial differential equation of higher order with boundary problem. It is very difficult to get its analytical solution, while we can use finite element method to get its numerical solution. The equation of element deflection could have the form [Rao, 2004]:

$$W(x) = \sum_{i=1}^{4} \mathbf{N_i}(x)\mathbf{q_i} \tag{12}$$

where q_i is the generalized coordinates. The shape functions $\mathbf{N_i}$ are equal to:

$$\begin{aligned} \mathbf{N_1} &= \tfrac{1}{l^3}(2x^3 - 3lx^2 + l^3) \\ \mathbf{N_2} &= \tfrac{1}{l^2}(x^3 - 2lx^2 + l^2 x) \\ \mathbf{N_3} &= \tfrac{1}{l^3}(3lx^2 - 2x^3) \\ \mathbf{N_4} &= \tfrac{1}{l^2}(x^3 - lx^2) \end{aligned} \tag{13}$$

Where l is the element length.

The kinetic and potential energies of the pipe element can be expressed by

$$T_1 = \frac{1}{2}\int_0^l (M+m)\left(\frac{\partial W}{\partial t}\right)^2 \mathrm{d}x = \frac{1}{2}\sum_e \dot{\mathbf{q}}^{\mathbf{T}}(M+m)\int_0^l \mathbf{N}^{\mathbf{T}}\mathbf{N}\,\mathrm{d}x\,\dot{\mathbf{q}} \tag{14}$$

$$V_1 = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 W}{\partial x^2}\right)^2 \mathrm{d}x = \frac{1}{2}\sum_e \mathbf{q}^{\mathbf{T}}\, EI \int_0^l \overline{\overline{\mathbf{N}}}^{\mathbf{T}}\overline{\overline{\mathbf{N}}}\,\mathrm{d}x\,\mathbf{q} \tag{15}$$

Where each prime sign that appear above the shape function symbol, i.e. "**N**", represent one-time derivative with respect to x-coordinate. Thus, mass ($\widehat{\mathbf{m}}$) and stiffness ($\hat{\mathbf{k}}_1$) matrices are equal to:

$$\widehat{\mathbf{m}} = \frac{(m+M)\,l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{16}$$

$$\hat{\mathbf{k}}_1 = \frac{2EI}{l^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix} \tag{17}$$

Over the length of elastic foundation, this adds the following term to the total potential energy:

$$V_2 = \frac{1}{2}\int_0^l k_o W^2 \, dx = \frac{1}{2}\sum_e \mathbf{q}^{\mathbf{T}} k_o \int_0^l \mathbf{N}^{\mathbf{T}}\mathbf{N} \, dx \, \mathbf{q} \tag{18}$$

We recognize the stiffness term in the above summation,

$$\hat{\mathbf{k}}_2 = k_o \int_0^l \mathbf{N}^{\mathbf{T}}\mathbf{N} \, dx \tag{19}$$

Thus, the foundation stiffness matrix equal to

$$\hat{\mathbf{k}}_2 = \frac{k_o l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{20}$$

The term ($MU^2 \frac{\partial^2 W}{\partial x^2}$) has a potential energy that can be represented in terms of displacement shape function derived for the pipe as

$$V_3 = \frac{1}{2}\int_0^l M\, U^2 \left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial x}\right) dx = \frac{1}{2}\sum_e \mathbf{q}^{\mathbf{T}} M U^2 \int_0^l \bar{\mathbf{N}}^{\mathbf{T}}\bar{\mathbf{N}} \, dx \, \mathbf{q} \tag{21}$$

The stiffness matrix that comes from flow around the deflected pipe is

$$\hat{\mathbf{k}}_3 = \frac{MU^2}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \tag{22}$$

It is important to clear that stiffness matrix $\hat{\mathbf{k}}_3$ leads to weaken the overall stiffness of the pipe system. The expression ($\mu \frac{\partial W}{\partial t}$) in eq. (11) has a dissipation energy as

$$\mathcal{R}_1 = \frac{1}{2}\int_0^l \mu \left(\frac{\partial W}{\partial t}\right)^2 dx = \frac{1}{2}\sum_e \dot{\mathbf{q}}^{\mathbf{T}} \mu \int_0^l \mathbf{N}^{\mathbf{T}}\mathbf{N} \, dx \, \dot{\mathbf{q}} \tag{23}$$

Leading to

$$\hat{\mathbf{C}}_1 = \mu \int_0^l \mathbf{N}^{\mathbf{T}}\mathbf{N} \, dx \tag{24}$$

The foundation viscous matrix equal to

$$\hat{\mathbf{C}}_1 = \frac{\mu l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{25}$$

While the expression ($2MU \frac{\partial^2 W}{\partial x \partial t}$) represent the Coriolis force, which causes the fluid in the pipe to whip, can be represented by dissipation energy as

$$\mathcal{R}_2 = \frac{1}{2}\int_0^l 2MU \left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial t}\right) dx = \frac{1}{2}\sum_e \mathbf{q}^{\mathbf{T}} \; 2MU \int_0^l \bar{\mathbf{N}}^{\mathbf{T}}\mathbf{N} \, dx \, \dot{\mathbf{q}} \tag{26}$$

This gives the unsymmetrical damping matrix

$$\hat{\mathbf{C}}_2 = \frac{MU}{30}\begin{bmatrix} -30 & -6l & -30 & 6l \\ 6l & 0 & -6l & l^2 \\ 30 & 6l & 30 & -6l \\ -6l & -l^2 & 6l & 0 \end{bmatrix} \tag{27}$$

DAMPED DYNAMIC EIGENVALUES

To analyze the dynamic eigenvalues of damped structure it must be transfer the governing equation to the state-space coordinates. The standard equation of motion in the finite element form is

$$(\mathbf{m} + \mathbf{M})\,\ddot{\mathbf{q}} + \mathbf{C}_{\mathbf{total}}\,\dot{\mathbf{q}} + \mathbf{k}_{\mathbf{total}}\,\mathbf{q} = \mathbf{0} \tag{28}$$

Where $\mathbf{C}_{\mathbf{total}} = \hat{\mathbf{C}}_1 + \hat{\mathbf{C}}_2$ and $\mathbf{k}_{\mathbf{total}} = \hat{\mathbf{k}}_1 + \hat{\mathbf{k}}_2 - \hat{\mathbf{k}}_3$. The substitution is, (Krodkiewski, [2008]:

$$\dot{\mathbf{q}} = \boldsymbol{\xi} \tag{29}$$

Resulting in the following set of equations

$$\dot{\mathbf{Z}} = \frac{dZ}{dt} = \mathbf{H}\,\mathbf{Z} \tag{30}$$

Where

$$\mathbf{Z} = \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\xi} \end{bmatrix} \tag{31}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{0} & \mathbf{II} \\ -(\mathbf{m} + \mathbf{M})^{-1}\mathbf{k}_{\mathbf{total}} & -(\mathbf{m} + \mathbf{M})^{-1}\mathbf{C}_{\mathbf{total}} \end{bmatrix} \tag{32}$$

Where **II** is a unity matrix.

Therefore, we can obtain the natural frequencies and mode shapes by solving the characteristic equation of

$$\lambda\,\mathbf{I} - \mathbf{H}\,\mathbf{Z} = \mathbf{0} \tag{33}$$

The solution of eigenvalue problem yields complex roots. The imaginary part of these roots represents the natural frequencies of damped system. The real part indicates the rate of decay of the free vibration.

LOGARITHMIC DECREMENT

A convenient way of determining the damping in a system is to measure the rate of decay of oscillation (the real part eigenvalue). The logarithmic decrement, δ, is the natural logarithm of the ratio of any two successive amplitudes in the same direction, where Y_1, and Y_2, are successive amplitudes, where (Beards, [1996])

$$\delta = \ln\left(\frac{Y_1}{Y_2}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \tag{34}$$

RESULT AND DISCUSSION

Figure (3-a) shows the relation between fluid velocity and the pipe frequency with various foundation stiffness of simply supported pipe mounted on viscoelastic foundation. As the foundation stiffness is increase, the natural frequency of the pipe is also increase. This behavior is taken place since the global stiffness of the system is increased. Furthermore, the relation between foundation stiffness and pipe frequency seems to be linear at constant fluid velocities as depicted in Figure (3-b). Figure (4**)** shows the effect of viscous damping coefficient of the foundation on the first natural frequency of the pipe system with different fluid velocities. As it is well known, the damped natural frequencies of damped system are smaller than it is for undamped one. Thus, an increase in the foundation viscous coefficient leads to reduce the dynamic properties of the pipe system. This behavior is not always true for damped system. Where the damped natural frequency of the lowest mode may be higher than the corresponding undamped frequency depending on the choice of damping matrix and the mode separation (Caughey and O'Kelly, [1961]). Figure 5 presents the relation between pipe length and critical flowing velocities for different foundation properties. As well known for a pipe without foundation, increasing the pipe length leads to reduce the pipe stiffness and raise its mass thus decreasing the critical flowing velocity. This behavior is differing for a pipe with viscoelastic foundation. Where there is a reduction in the critical flowing velocities with increasing the pipe length then at some pipe length the critical velocity start to rise, then after reducing again and continue with compacted values. This convexity in the curve mainly caused by the foundation stiffness and this behavior is completely agree with the study that done by Chen [1991]. Moreover, when increasing foundation damping, the critical velocities exhibit more reduction in their values than it for small damping does. This behavior is caused by increasing the overall damping of the system that leading to decrease its damped natural frequency. Thus, we can say that, in viscoelastic foundation, damping induces destabilization effect, while foundation stiffness leads to stabilize the pipe. It has thus found that the criterion for global instability as the length is increased becomes closely related to the local properties of the waves in the pipe (Doared and E. De Langre, [2002]). It is important to record that at some ranges of pipe length, the foundation viscosity effect seems extreme and obvious. Figure (6) shows the effect of fluid velocity on the system's damping ratio for two different foundation viscosities. The damping ratio will decrease monotonically with increasing fluid velocity. This event is mainly caused by decreasing the rate decay of pipe vibration. Figures (7-a and b) show the percentage errors in the predicted natural frequencies of the pipe system with neglecting the Coriolis component for different pipe lengths and fluid densities respectively. From these figures, a pipe with larger length and higher fluid density has the biggest frequency percentage error. Furthermore, increasing the flowing fluid velocity leads to increase the percentage error.

CONCLUSIONS

The effects of a viscoelastic foundation on the stability of a fluid conveying pipe were analyzed by using the extension of Winkler foundation model. The problem was analyzed numerically using finite element method. Some interesting conclusions have been drawn, as follows:

(1) The foundation stiffness leads to increase the fluid critical velocity , while foundation damping decreases it.

(2) There are ranged values of pipe length for each foundation properties that seems to be more affected by the foundation characteristics.
(3) Coriolis component still play a major role in the dynamic behavior of pipe especially with larger length and heavier fluid.
(4) The damping ratio of the system is decreased monotonically with increasing the fluid velocity.

ACKNOWLEDGEMENTS
The author would like to thank University of Babylon for the technical support.

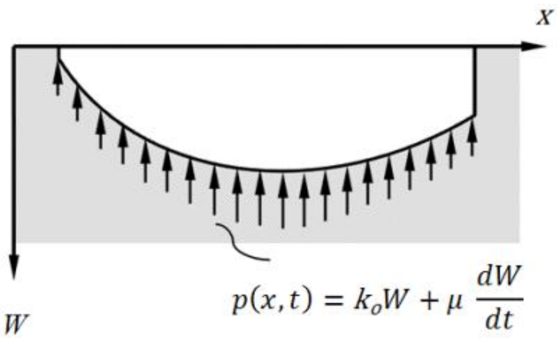


Figure 1: Modified Winkler Foundation Model.

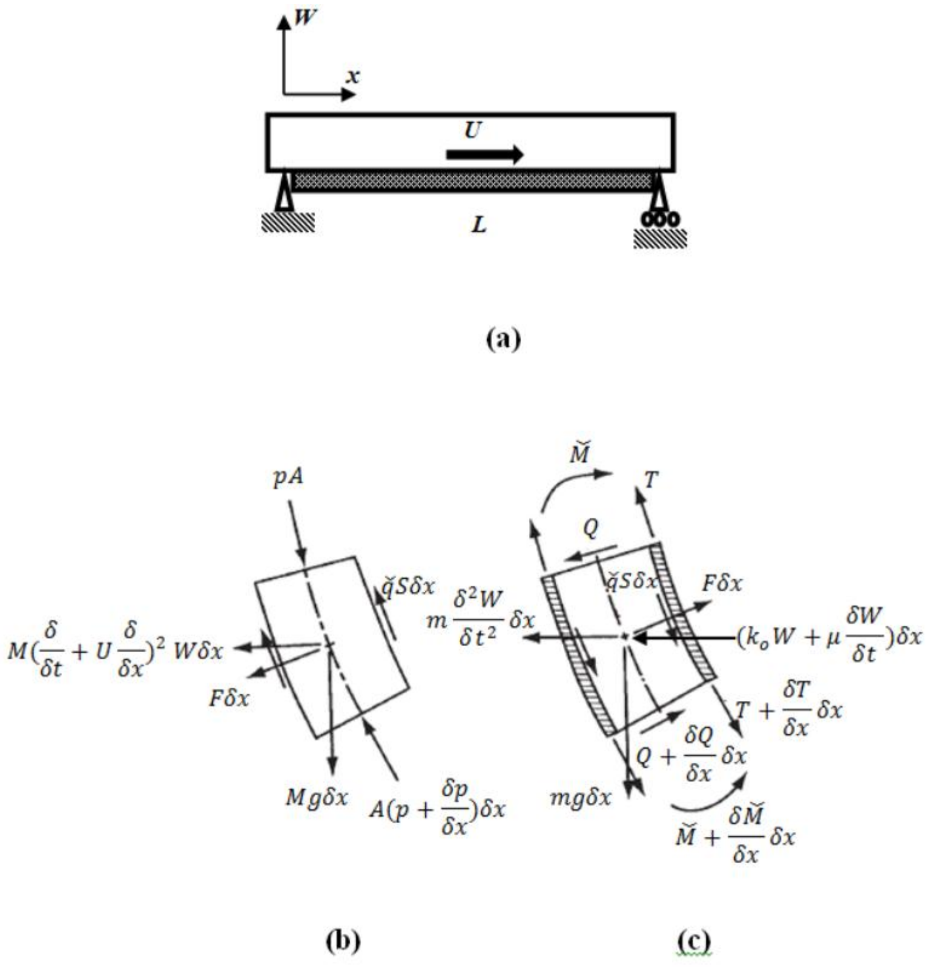


Fig. 2: **(a)** Simply supported pipe on viscoelastic foundation **(b)** Forces on fluid element, and **(c)** Forces and moments on pipe element.

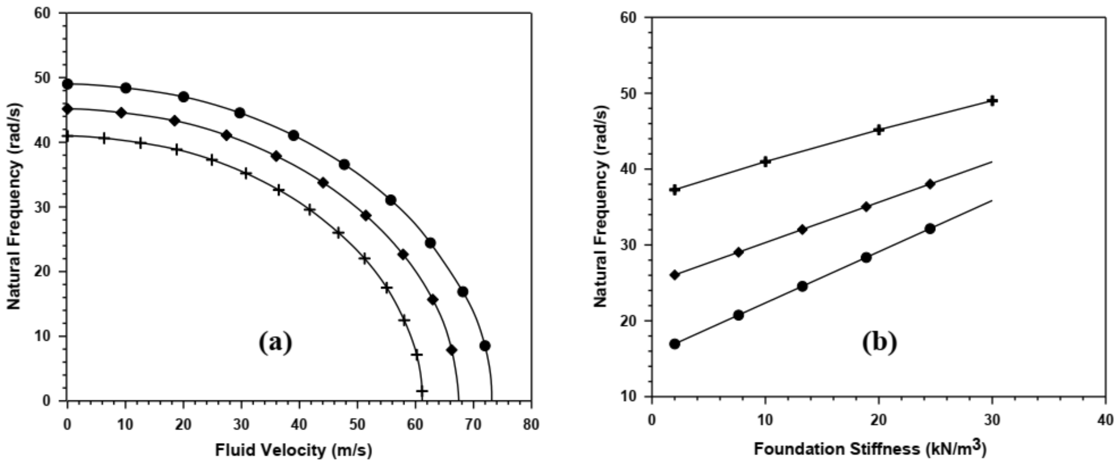


Fig. 3: Effect of foundation stiffness on the natural frequency of the pipe at different fluid velocities. Pipe length is (2 m), fluid density is (1000 kg/m^3), pipe density is (8000 kg/m^3), pipe thickness is (0.001 m) , outer diameter of the pipe is (0.01 m), elastic modulus of pipe is (207 GPa) and foundation damping coefficient , μ_v, is equal to (100 N.s/m^3). **(a)** Label symbols are ● , k_v=10 kN/m^3; ♦, k_v=20 kN/m^3 ; +, k_v=30 kN/m^3 .**(b)** Label symbols are ● , U=50 m/s; ♦, U=40 m/s; +, U=0 m/s.

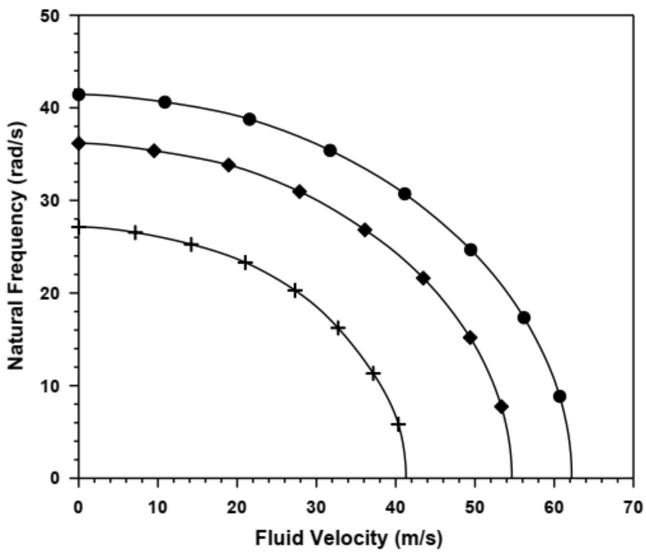


Fig. 4: Effect of foundation viscous damping on the natural frequency of the pipe at different fluid velocities. Pipe length is (2 m), fluid density is (1000 kg/m^3), pipe density is (8000 kg/m^3), pipe thickness is (0.001 m) , outer diameter of the pipe is (0.01 m), elastic modulus of pipe is (207 GPa) and foundation stiffness coefficient , k_v, is equal to (20 kN /m^3). Label symbols are ● , μ_v=1 kN.s/m^3; ♦, μ_v =1.5 kN.s/m^3 ; +, μ_v =2 kN.s/m^3.

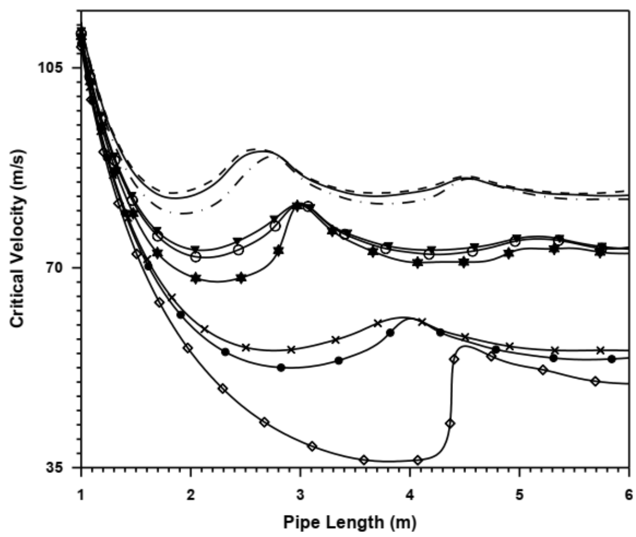


Fig. 5: Effect of pipe length on the critical fluid velocity for different mounting conditions. Pipe density is (8000 kg/m^3) ,fluid density is (1000 kg/m^3), outer diameter of the pipe is (0.01m)), pipe thickness is (0.001 m), elastic modulus of pipe is (207 GPa). Label symbols are: dash line, k_v=50 kN/m^3 and μ_v =0; solid line, k_v=50 kN/m^3 and μ_v =500 N.s/m^3; centerline, k_v=50 kN/m^3 and μ_v =1000 N.s/m^3 ; ▸, k_v=30 kN/m^3 and μ_v =0; ○, k_v=30 kN/m^3 and μ_v =500 N.s/m^3; ✱, k_v=30 kN/m^3 and μ_v =1000 N.s/m^3; ×, k_v=30 kN/m^3 and μ_v =0; ●, k_v=10 kN/m^3 and μ_v =500 N.s/m^3; ◇, k_v=10 kN/m^3 and μ_v =1000 N.s/m^3.

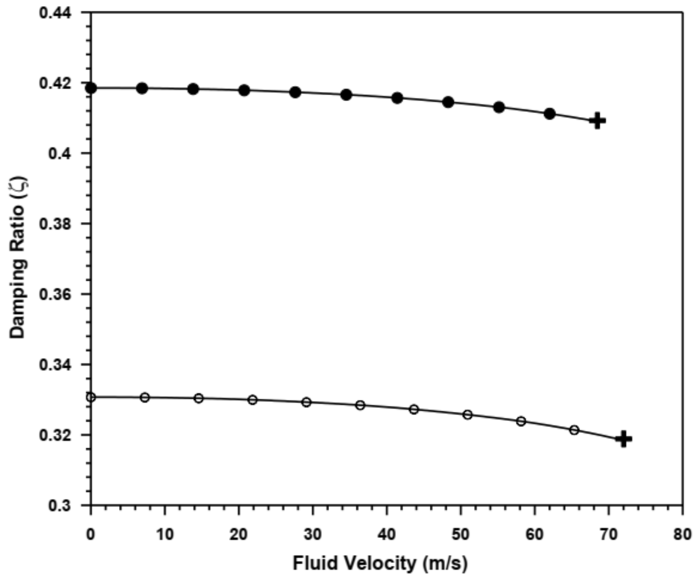
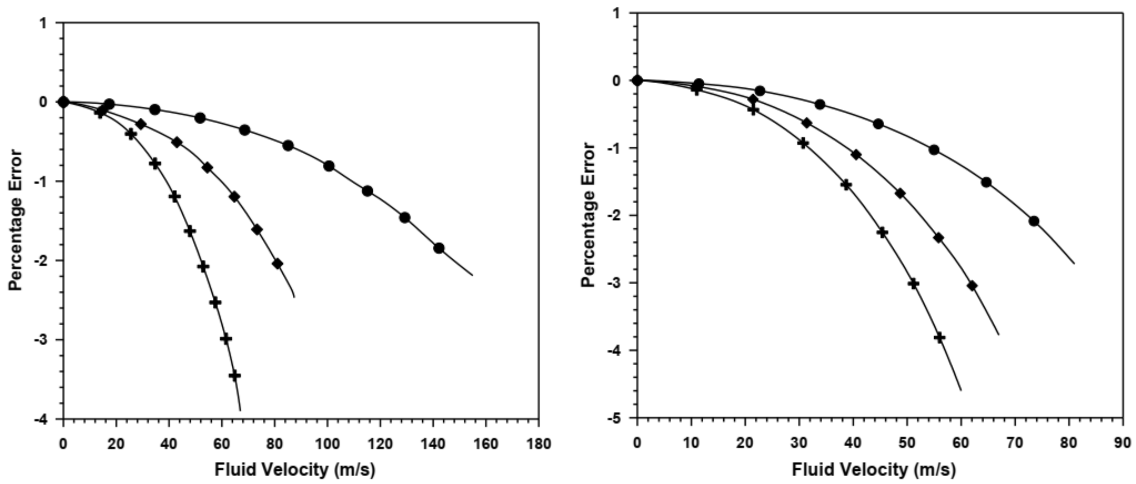


Fig. (6): Effect of fluid velocity on the system's damping ratio. Pipe density is (8000 kg/m^3) ,fluid density is (1000 kg/m^3), outer diameter of the pipe is (0.01 m)), pipe thickness is (0.001 m), elastic modulus of pipe is (207 GPa), pipe length is (2 m). Label symbols are: ○, k_v=30 kN/m^3 and μ_v =1000 N.s/m^3; ●, k_v=30 kN/m^3 and μ_v =500 N.s/m^3; ✚, critical velocity.



(a) **(b)**

Fig.7: Effect of whether considering Coriolis force in (**a**) different pipe lengths, Label symbols are ●, pipe length is 0.7 m ; ♦, pipe length is 1.3 m ; +, pipe length is 2 m (**b**) different fluid densities, Label symbols are ●, fluid density is 680 kg/m^3 ; ♦, fluid density is 1000 kg/m^3 ; +, fluid density is 1260 kg/m^3. The parameters are : pipe density is (8000 kg/m^3), outer diameter of the pipe is (0.01 m), elastic modulus of pipe is (207 GPa), foundation stiffness coefficient, k_v, is (20 kN /m^3) and foundation damping coefficient, μ_v, is (1000 N.s/m^3).

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